Magnetohydrodynamics of GRB jets

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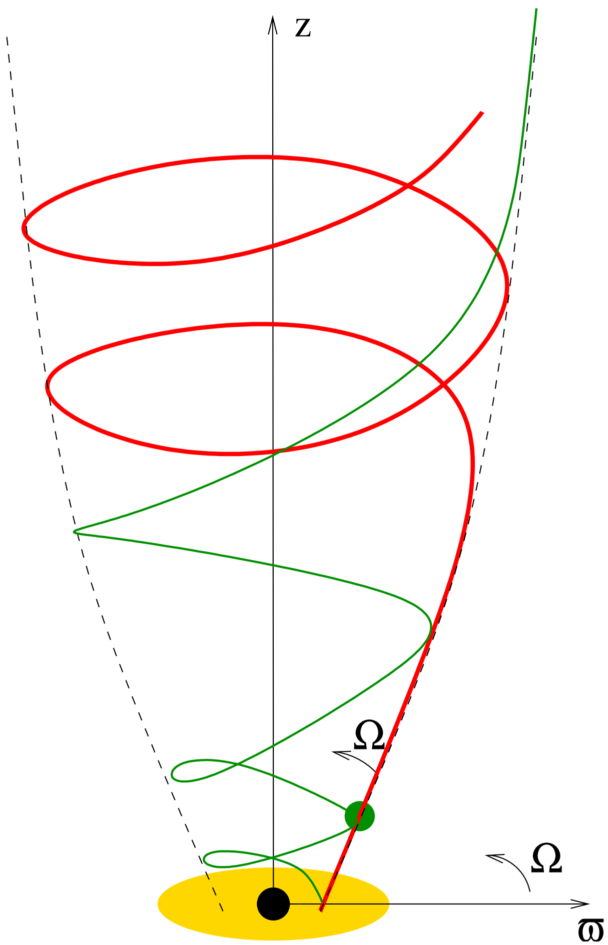
Outline

- basic questions for magnetized flows
- answers from analytical – numerical models
- implications for GRB jets

Magnetized outflows

A rotating source (disk or star) creates an axisymmetric outflow



- Assume ideal magnetohydrodynamics
- Ejected mass per time $\dot{M}$
- Extracted energy per time $\dot{\mathcal{E}}$

mainly in the form of Poynting flux

$$\dot{\mathcal{E}} = \frac{c}{4\pi} \underbrace{\frac{r\Omega}{c} B_p}_{E} B_\phi \times (\text{ area }) \approx \frac{c}{2} B^2 r^2$$

$B \sim 10^{14}$G for disk-driven, or, $B \sim 10^{15}$G for black hole-driven, give $\dot{\mathcal{E}} \sim 10^{50}$ergs/s

- The $\mu \equiv \dot{\mathcal{E}}/\dot{M}c^2$ crucial parameter

(it gives the maximum possible Lorentz factor of the flow)

- MHD: outflowing matter (velocity, density, pressure)

  + large scale electromagnetic field

Basic questions

☞ bulk acceleration

- thermal (due to ∇P) $\longrightarrow$ velocities up to C_s
- magnetocentrifugal $\longrightarrow$ velocities up to $V_{\phi i}$
- relativistic thermal (thermal fireball) gives $\gamma \sim \left(\frac{\text{enthalpy}}{\text{mass} \times c^2}\right)_{\rm i}$.
- magnetic ($\boldsymbol{J} \times \boldsymbol{B}$ force)
  acceleration efficiency $\gamma_\infty/\mu =$?
  terminal γ_∞ ?

☞ collimation

hoop-stress + electric force

degree of collimation ?

jet opening angle ?

Key steps

- Michel 1969:
  assuming monopole flow (crucial) $\rightarrow$ inefficient acceleration with $\gamma_\infty \approx \mu^{1/3} \ll \mu$

- Li, Chiueh & Begelman 1992:
  cold self-similar model $\rightarrow \gamma_\infty \approx \mu/2$ (50% efficiency)

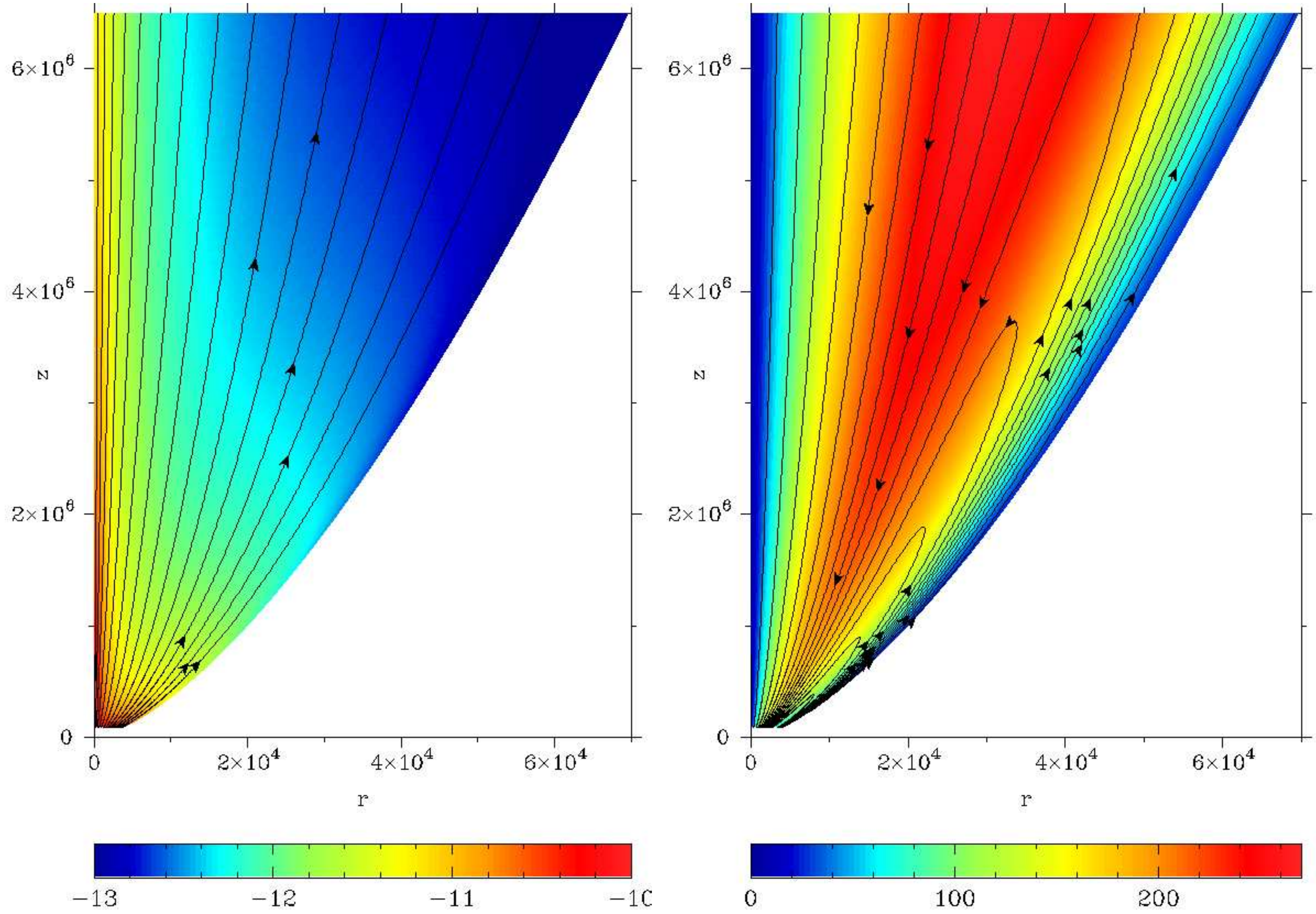
- Vlahakis & Königl 2003:
  generalization of the self-similar model (including thermal and radiation effects) $\rightarrow \gamma_\infty \approx \mu/2$ (50% efficiency)

- Vlahakis 2004:
  complete asymptotic transfield force-balance
  explain why Michel's model fails, describe why the flow-shape (collimation) controls the acceleration

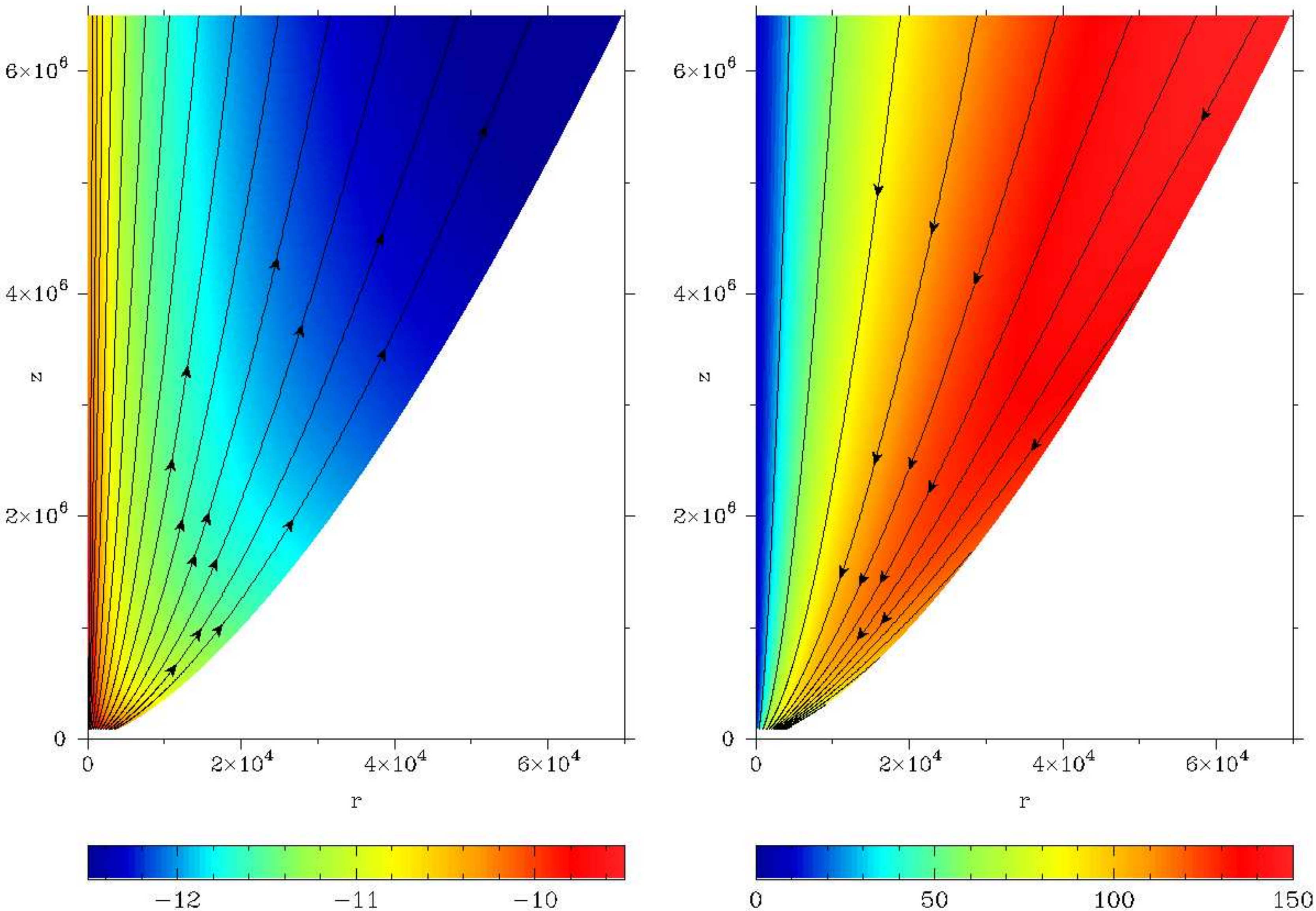
Key steps (cont'd)

- Komissarov, Barkov, Vlahakis & Königl 2007 and Komissarov, Vlahakis, Königl & Barkov 2009:
  possible for the first time to simulate high γ MHD flows and follow the acceleration up to the end
  + analytical scalings
  + role of causality, role of external pressure
- Tchekhovskoy, McKinney & Narayan, 2009: simulations of nearly monopolar flow (although it was covered in Komissarov et al 2009, the analysis here is more detailed)
  Even for nearly monopolar flow the acceleration is efficient near the rotation axis
- Lyubarsky 2009:
  generalization of the analytical results of Vlahakis 2004 and Komissarov et al 2009

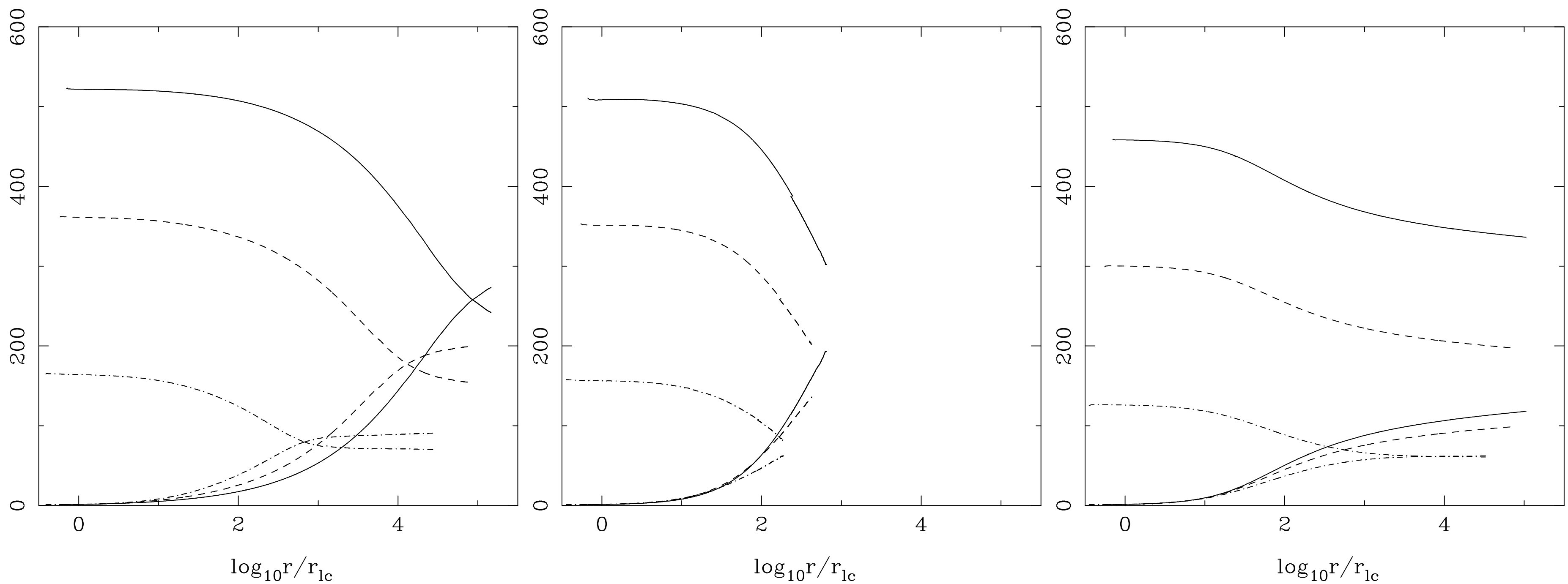
Komissarov, Vlahakis, Königl, & Barkov 2009



left: density/field lines, right: Lorentz factor/current lines (wall shape $z \propto r^{1.5}$)
Differential rotation → slow envelope



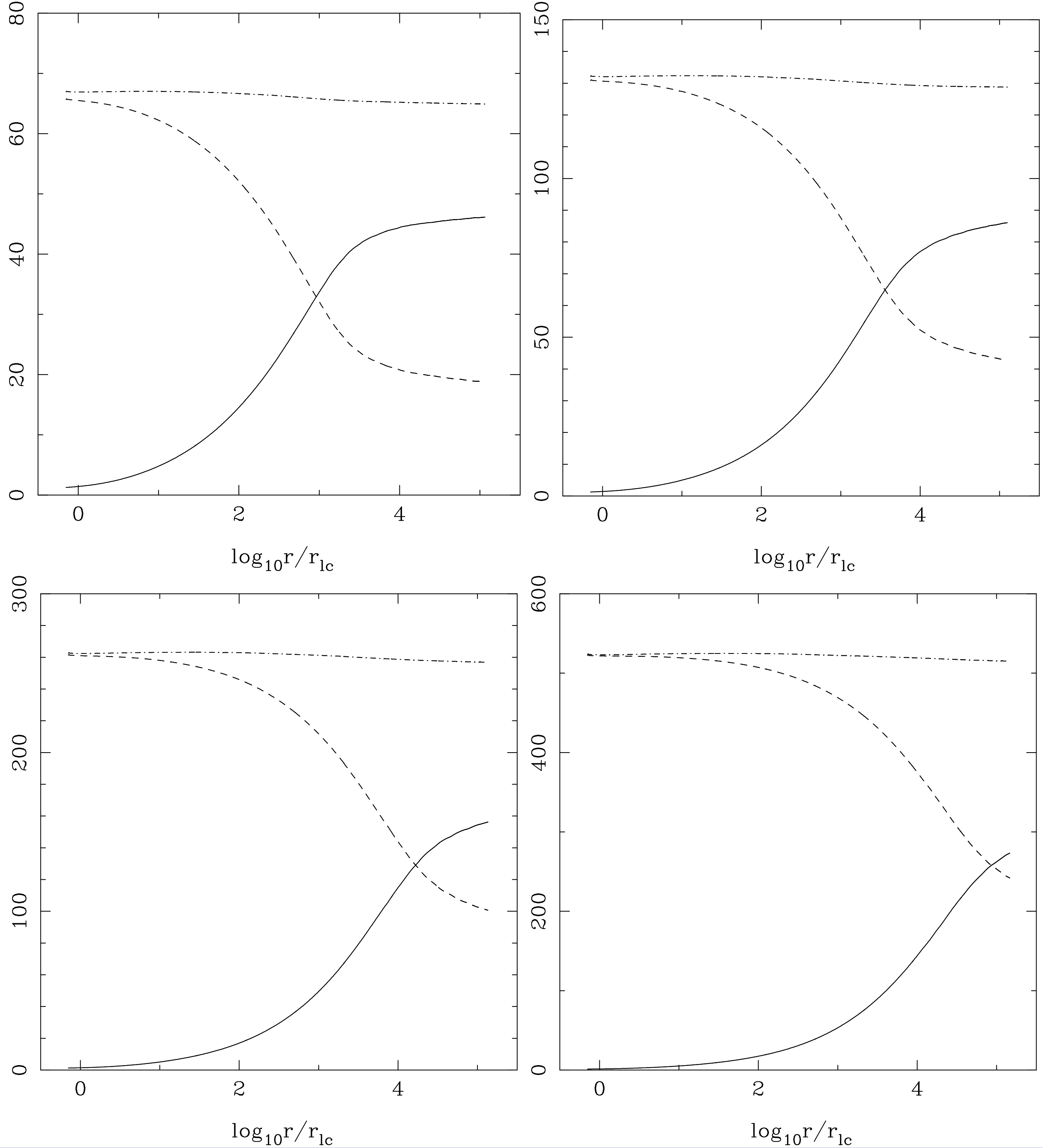
Uniform rotation $\rightarrow \gamma$ increases with r



γ and $\gamma\sigma$ for wall-shapes:
$z \propto r^{3/2}$ (left), $z \propto r^2$ (middle), $z \propto r$ (right)

In parabolic, Lorentz factor $\gamma \sim z/r \propto r^{1/2} \propto R^{1/3}$ (left)
and $\gamma \sim z/r \propto r \propto R^{1/2}$ (middle)
efficiency more than 50%

In the conical $\gamma \sim r\Omega/c$, but smaller efficiency, except very close to the axis



log10r/rlc
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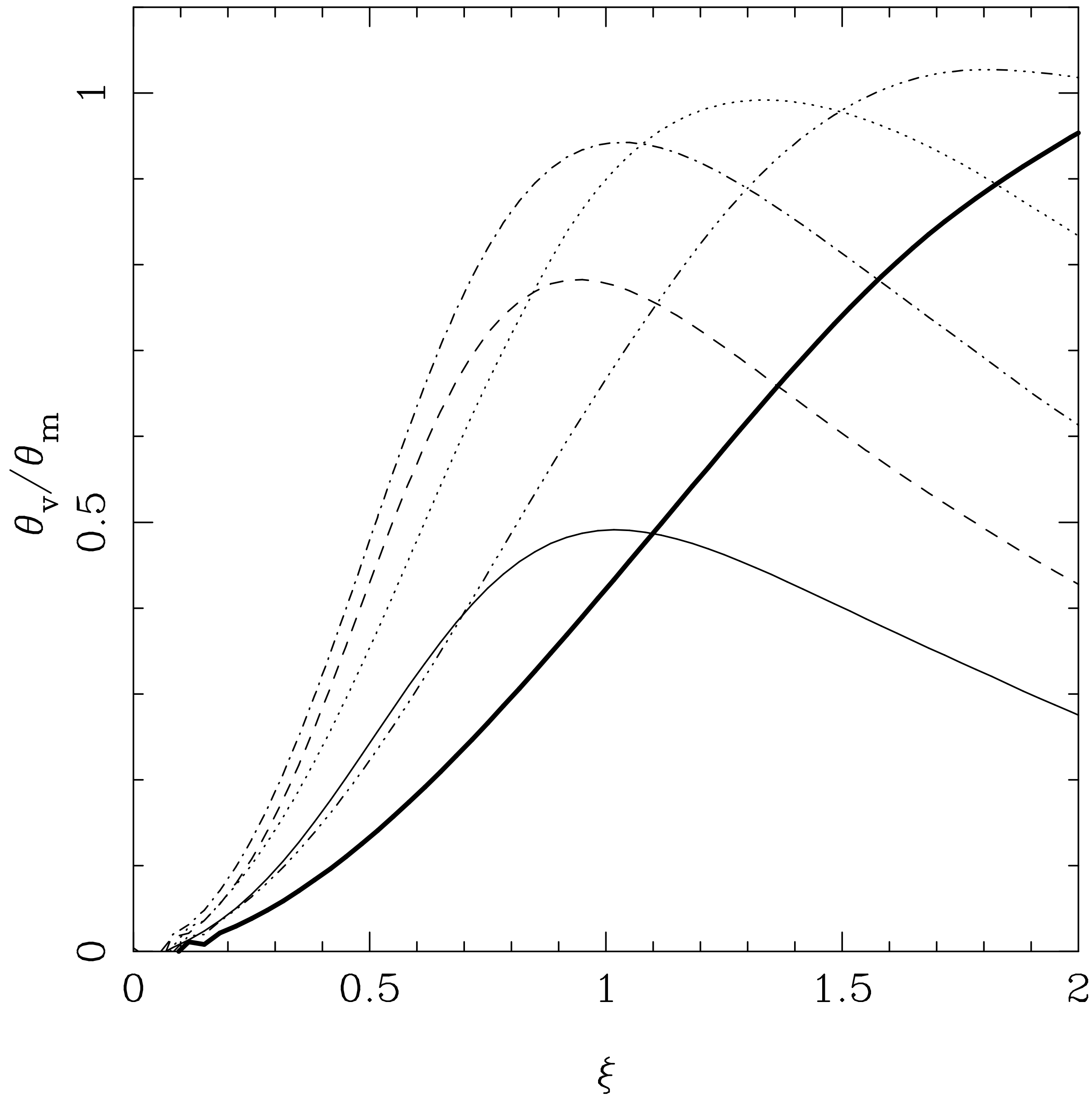
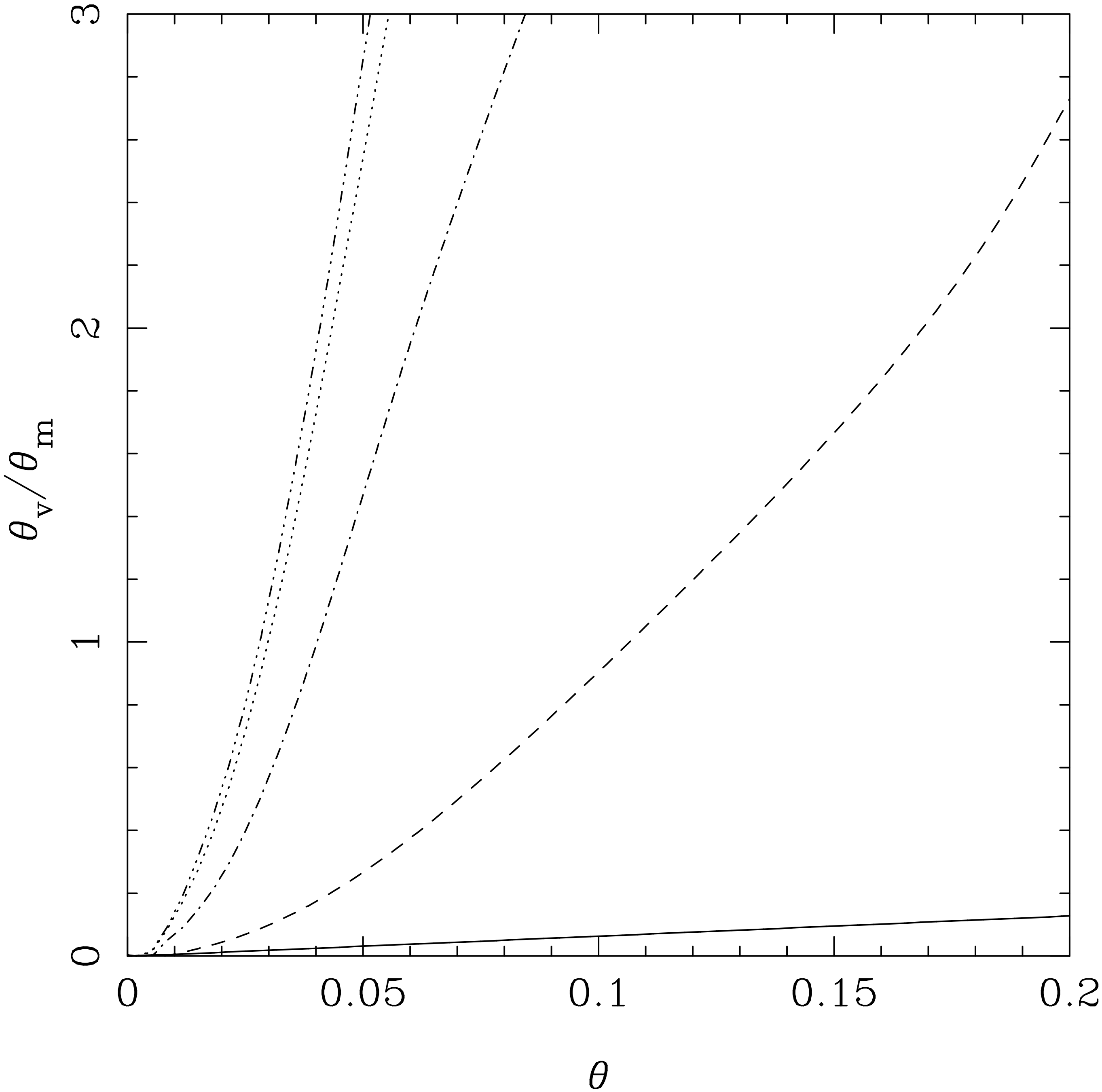
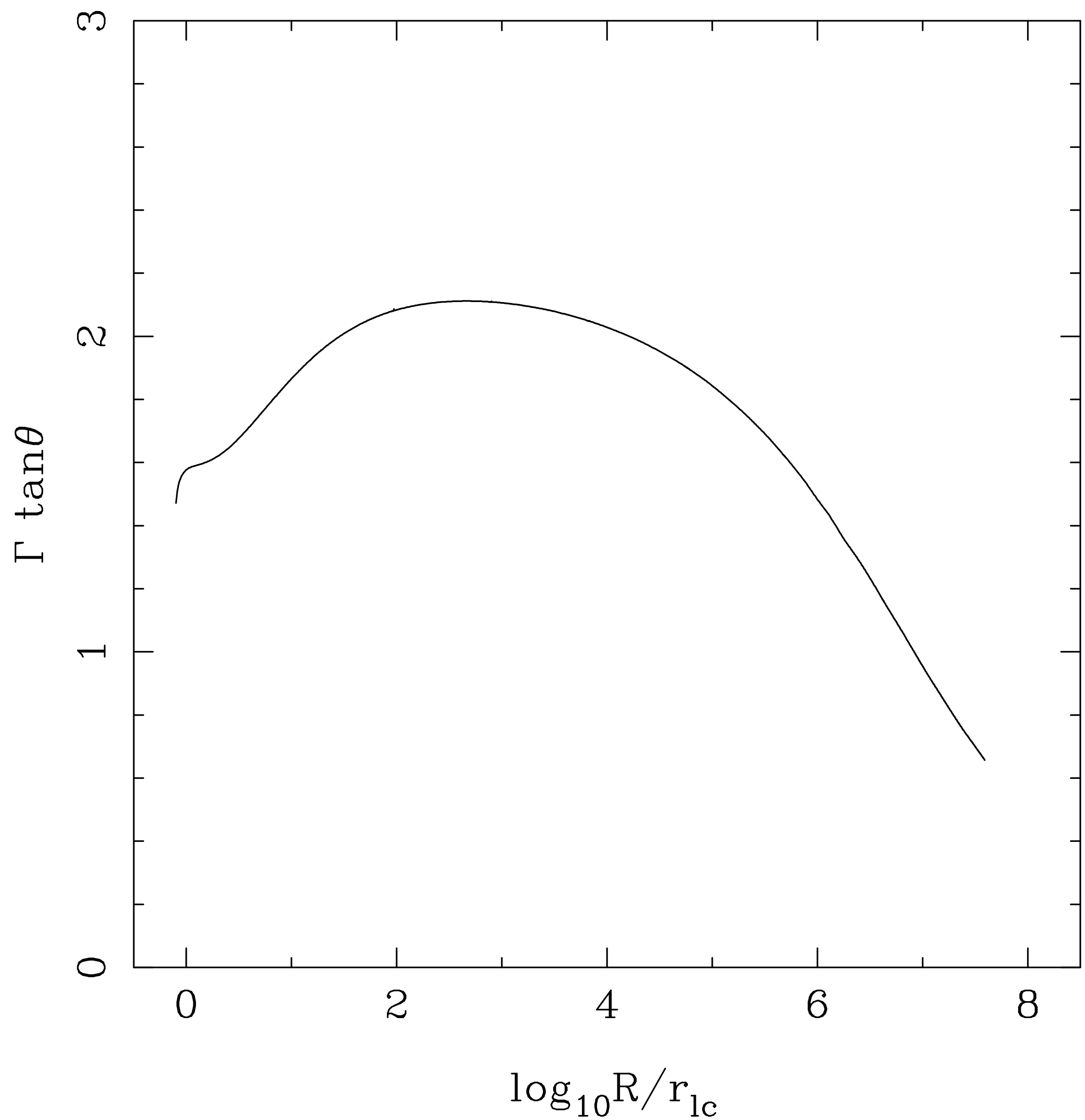


θ_v = jet opening angle, θ_m = Mach-cone opening angle

causal connection → collimation → acceleration



in the conical model $\theta_v < \theta_m$ only close to the axis



$\gamma \frac{dr}{dz} \sim 1 \Rightarrow \gamma \sim dz/dr \sim 1/\theta_{\rm v}$

no break in parabolic flows! (or only weak break at very early times)

Linear *vs* Power-law acceleration

⋆ The wall-shape $z \propto r^a$ and thus the acceleration law (linear $\gamma \propto r$ or power-law $\gamma \sim z/r \propto r^{a-1}$) is controlled by the external pressure $p_{\rm ext} \propto z^{-\alpha_p}$:

- if $\alpha_p < 2$ (the pressure drops slower than z^{-2}) then $a > 2$ (shape more collimated than $z \propto r^2$).
- if $\alpha_p = 2$ then $1 < a \le 2$.
- if $\alpha_p > 2$ (pressure drops faster than z^{-2}) then $a = 1$.

⋆ Linear acceleration for weak external pressure efficient only close to axis.
$\gamma\theta_{\rm v} > 1 \rightarrow$ break

⋆ Power-law acceleration is efficient.
$\gamma\theta_{\rm v} \sim 1$

Discussion

⋆ Magnetic driving provides a viable explanation of the dynamics of GRB jets

- depending on the external pressure,
  collimation to parabolic shape $z \propto r^a$ – with $\gamma \sim z/r \propto r^{a-1}$,
  or conical shape $z \propto r$ with $\gamma \propto r$
- bulk acceleration up to Lorentz factors $\gamma_\infty \gtrsim 0.5\frac{\mathcal{E}}{Mc^2}$
  (in conical flows only near the axis)

⋆ next steps:

- calculation of $\gamma\theta_{\rm v}$ in non-parabolic (free boundary) jets with power-law acceleration
- application to the external pressure in collapsar and binary merger models?
  breaks?