Neutron Star Equations of State

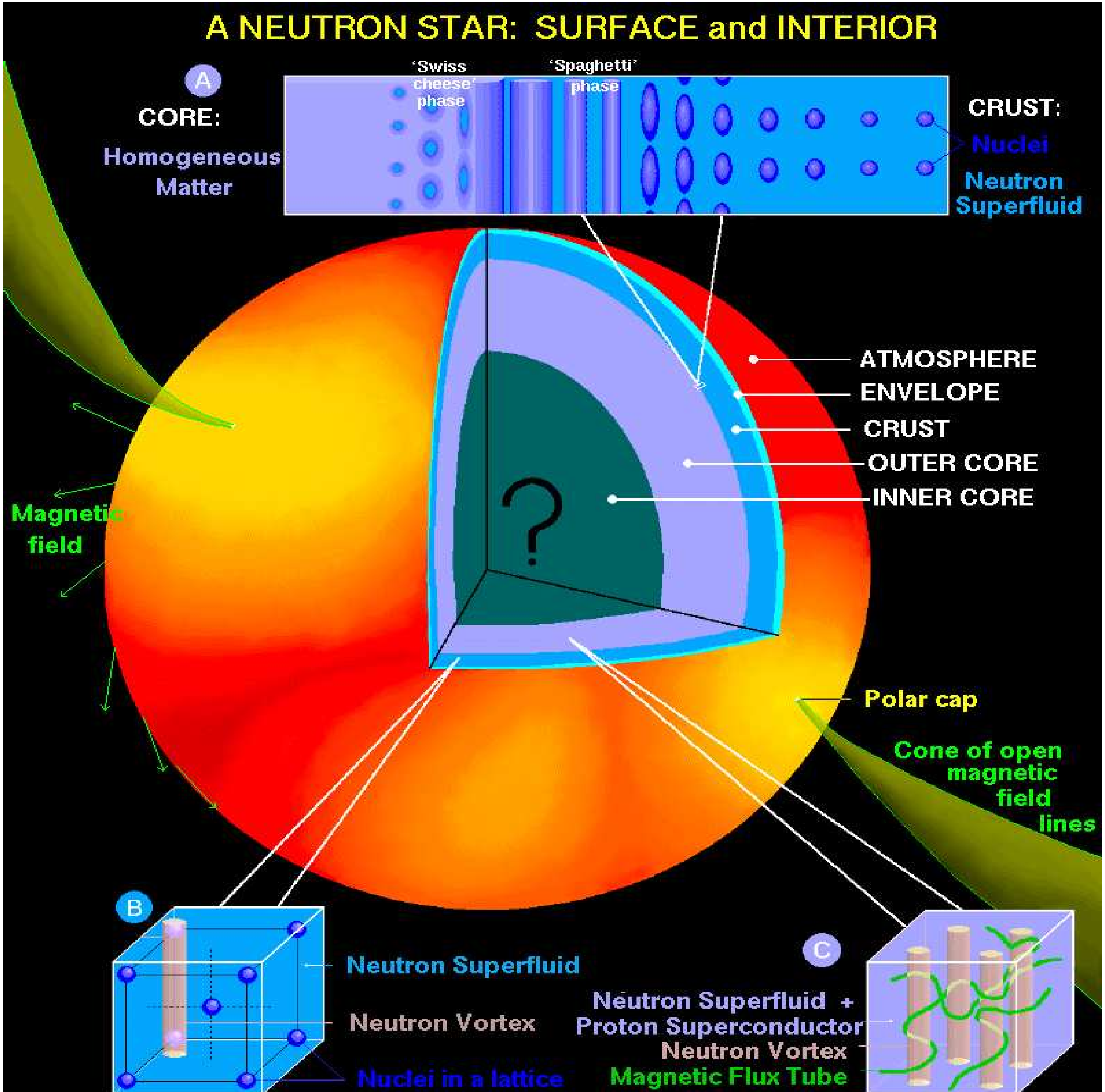
James M. Lattimer

lattimer@astro.sunysb.edu

Department of Physics & Astronomy
Stony Brook University

Neutron Stars and the Equation of State

- Extreme Properties
- Pulsar Constraints – Rotation and Mass
- Pressure–Radius Correlation
- Nuclear Symmetry Energy
- Nuclear Structure Constraints
- Observational Mass and Radius Constraints
- Inverting the TOV Equations



Credit: Dany Page, UNAM

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\begin{aligned}\frac{dp}{dr} &= -\frac{G}{c^2}\frac{(m+4\pi pr^3)(\epsilon+p)}{r(r-2Gm/c^2)} \\ \frac{dmc^2}{dr} &= 4\pi\epsilon r^2\end{aligned}$$

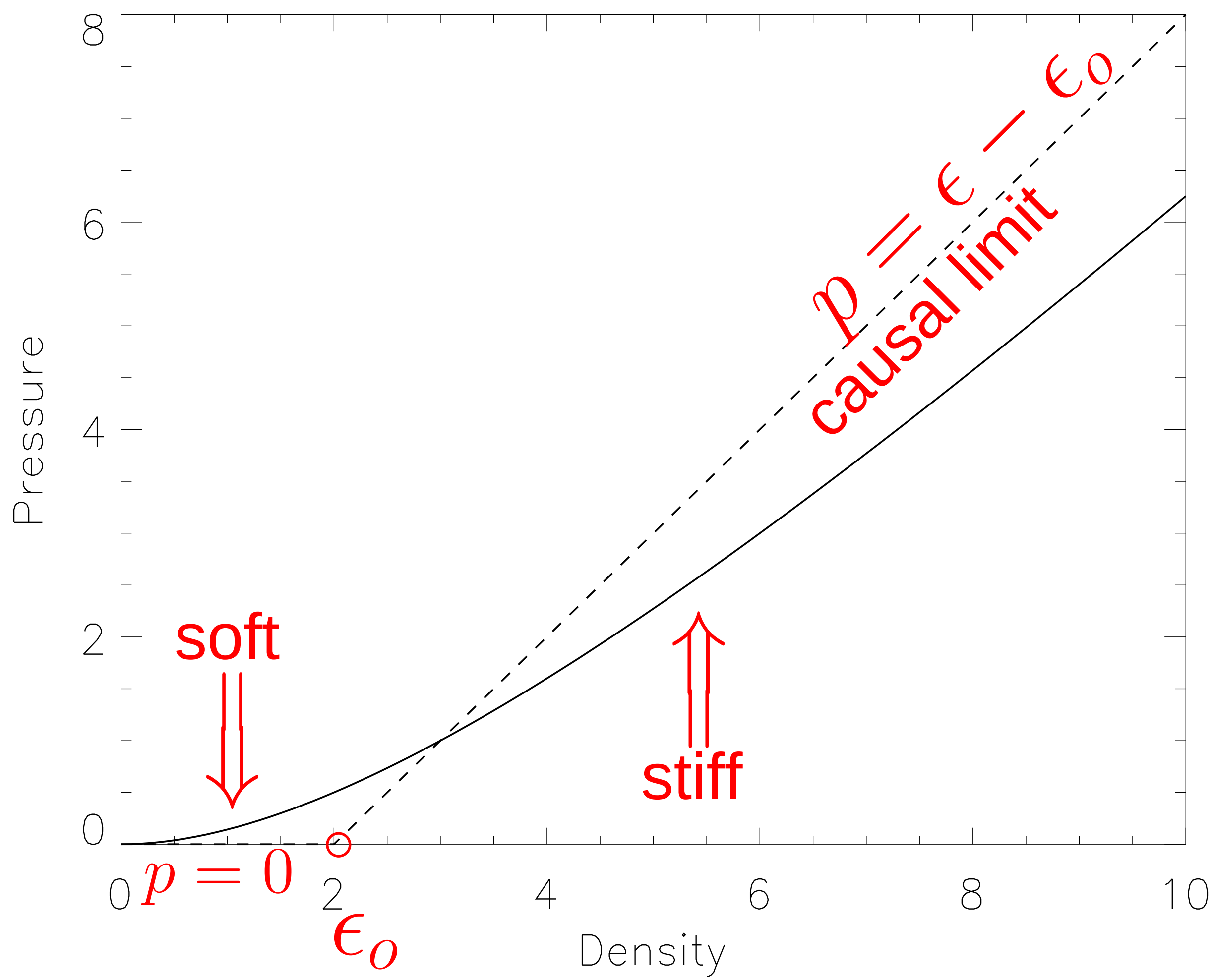
p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

Maximum Mass, Minimum Period

Theoretical limits from GR and causality

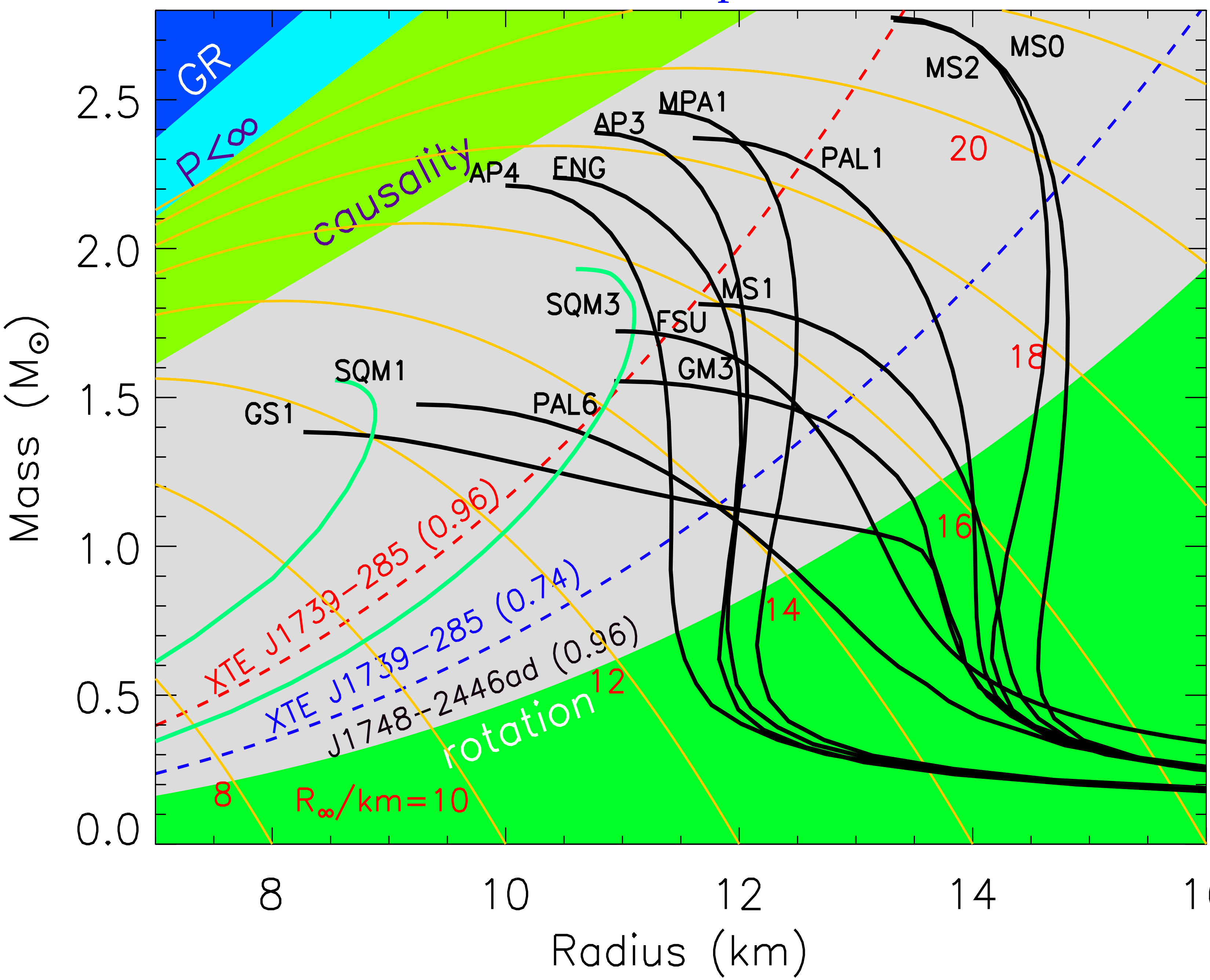
- $M_{max} = 4.2(\epsilon_s/\epsilon_0)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq (0.74 \pm 0.03)(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms (empirical)

  Lattimer & Prakash (2004)
- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

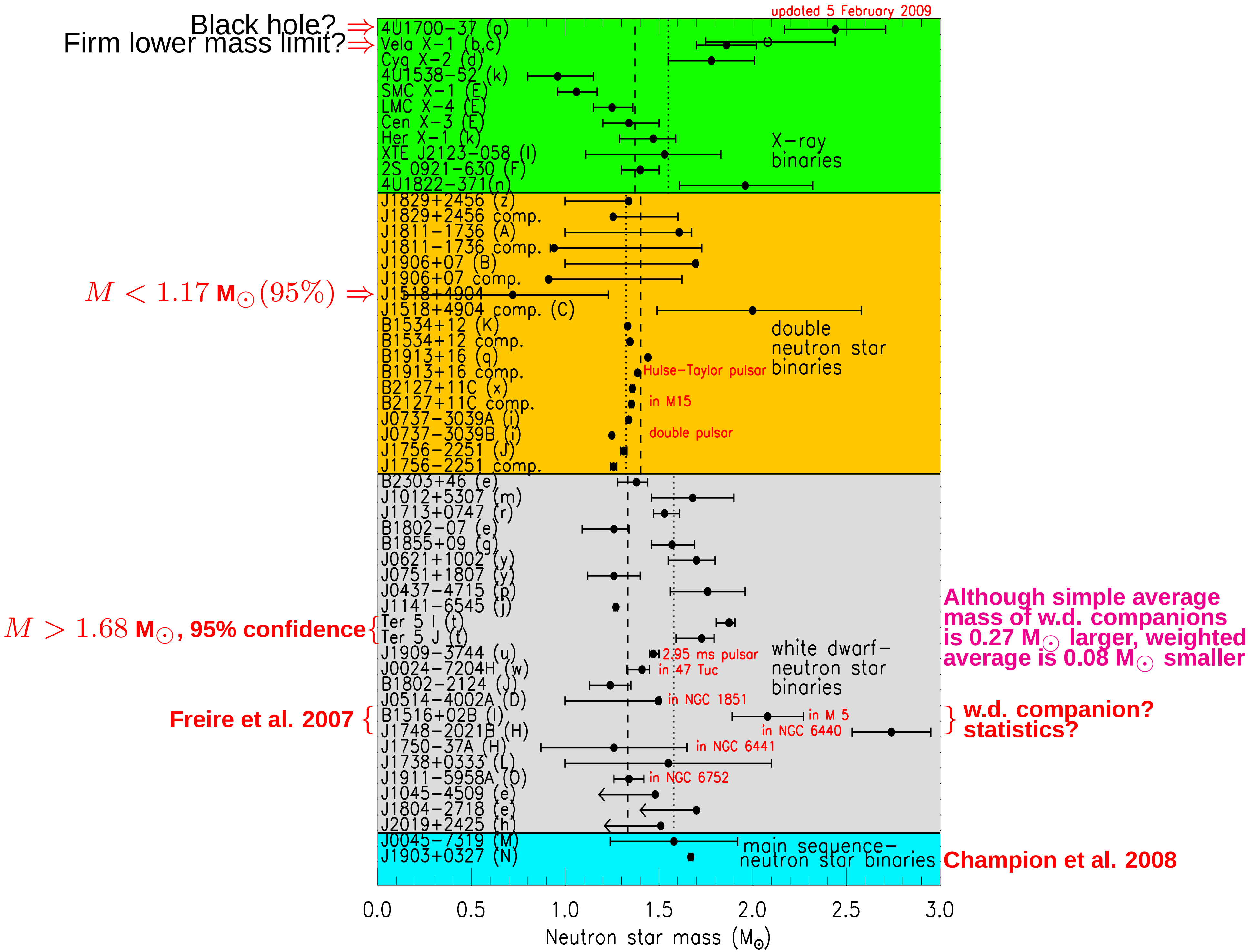
Constraints from Pulsar Spins



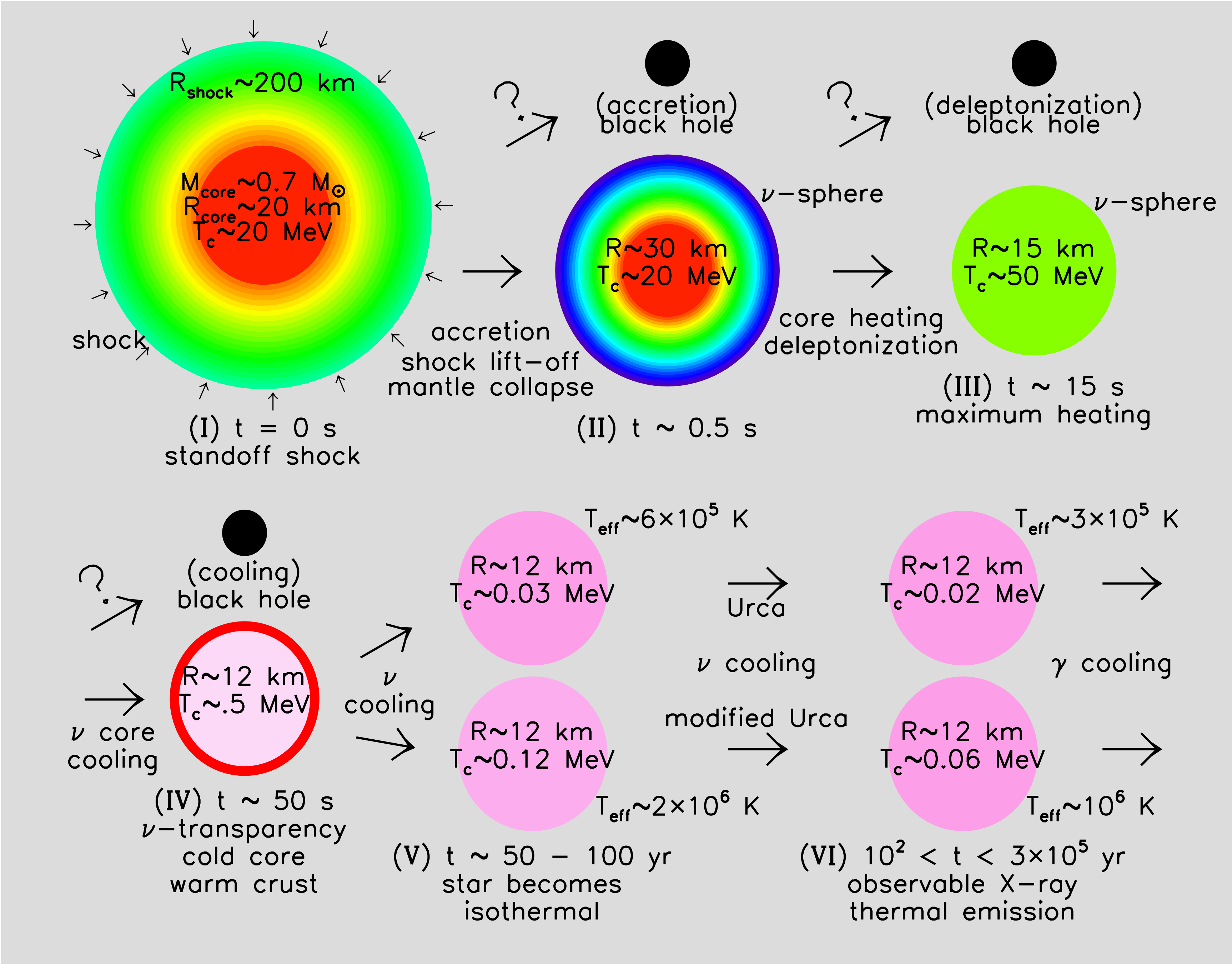
XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

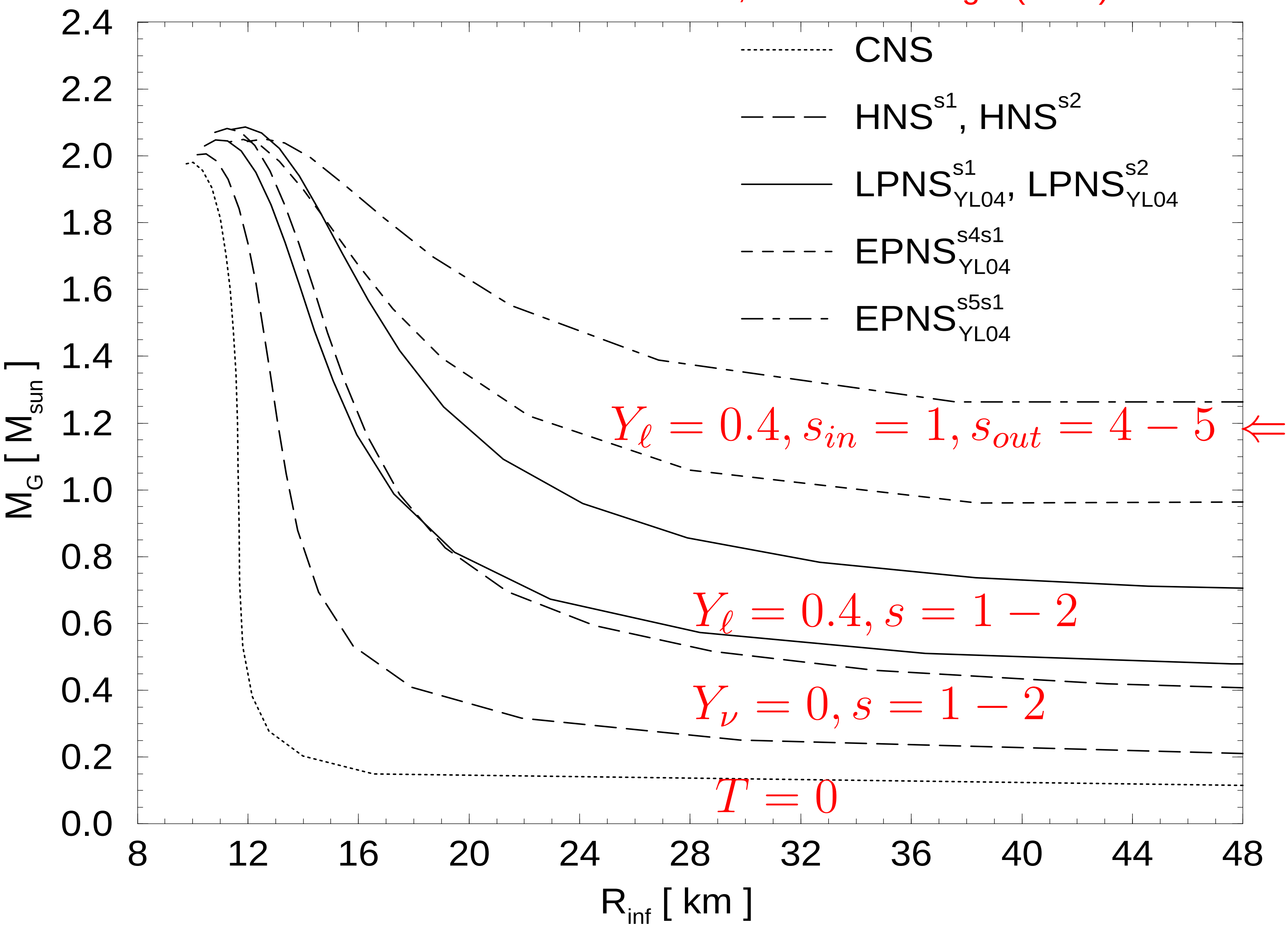

updated 5 February 2009
Black hole? ⇒
Firm lower mass limit? ⇒
4U1700−37 (a)
Vela X−1 (b,c)
Cyg X−2 (d)
4U1538−52 (k)
SMC X−1 (E)
LMC X−4 (E)
Cen X−3 (E)
Her X−1 (k)
XTE J2123−058 (l)
2S 0921−630 (F)
4U1822−371 (n)
X−ray binaries
J1829+2456 (z)
J1829+2456 comp.
J1811−1736 (A)
J1811−1736 comp.
J1906+07 (B)
J1906+07 comp.
$M < 1.17$ M$_\odot$ (95%) ⇒
J1518+4904
J1518+4904 comp. (C)
B1534+12 (K)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
Hulse−Taylor pulsar
B2127+11C (x)
B2127+11C comp.
in M15
J0737−3039A (i)
J0737−3039B (i)
double pulsar
J1756−2251 (J)
J1756−2251 comp.
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (g)
J0621+1002 (y)
J0751+1807 (y)
J0437−4715 (p)
J1141−6545 (j)
$M > 1.68$ M$_\odot$, 95% confidence {
Ter 5 I (t)
Ter 5 J (t)
J1909−3744 (u)
2.95 ms pulsar
J0024−7204H (w)
in 47 Tuc
B1802−2124 (J)
J0514−4002A (D)
in NGC 1851
Freire et al. 2007 {
B1516+02B (I)
in M 5
J1748−2021B (H)
in NGC 6440
J1750−37A (H)
in NGC 6441
J1738+0333 (L)
J1911−5958A (O)
in NGC 6752
J1045−4509 (e)
J1804−2718 (e)
J2019+2425 (h)
white dwarf−neutron star binaries
Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.08 M$_\odot$ smaller
} w.d. companion? statistics?
J0045−7319 (M)
J1903+0327 (N)
main sequence−neutron star binaries
Champion et al. 2008
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass (M$_\odot$)

Proto-Neutron Stars



Effective Minimum Masses

Strobel, Schaab & Weigel (1999)



Neutron Star Matter Pressure and the Radius

$$p \simeq K\epsilon^{1+1/n}$$

$$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$$

$$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$$

$$R \propto p_*^{1/2}\epsilon_*^{-1} M^0$$

$$(1 < \epsilon_*/\epsilon_0 < 2)$$

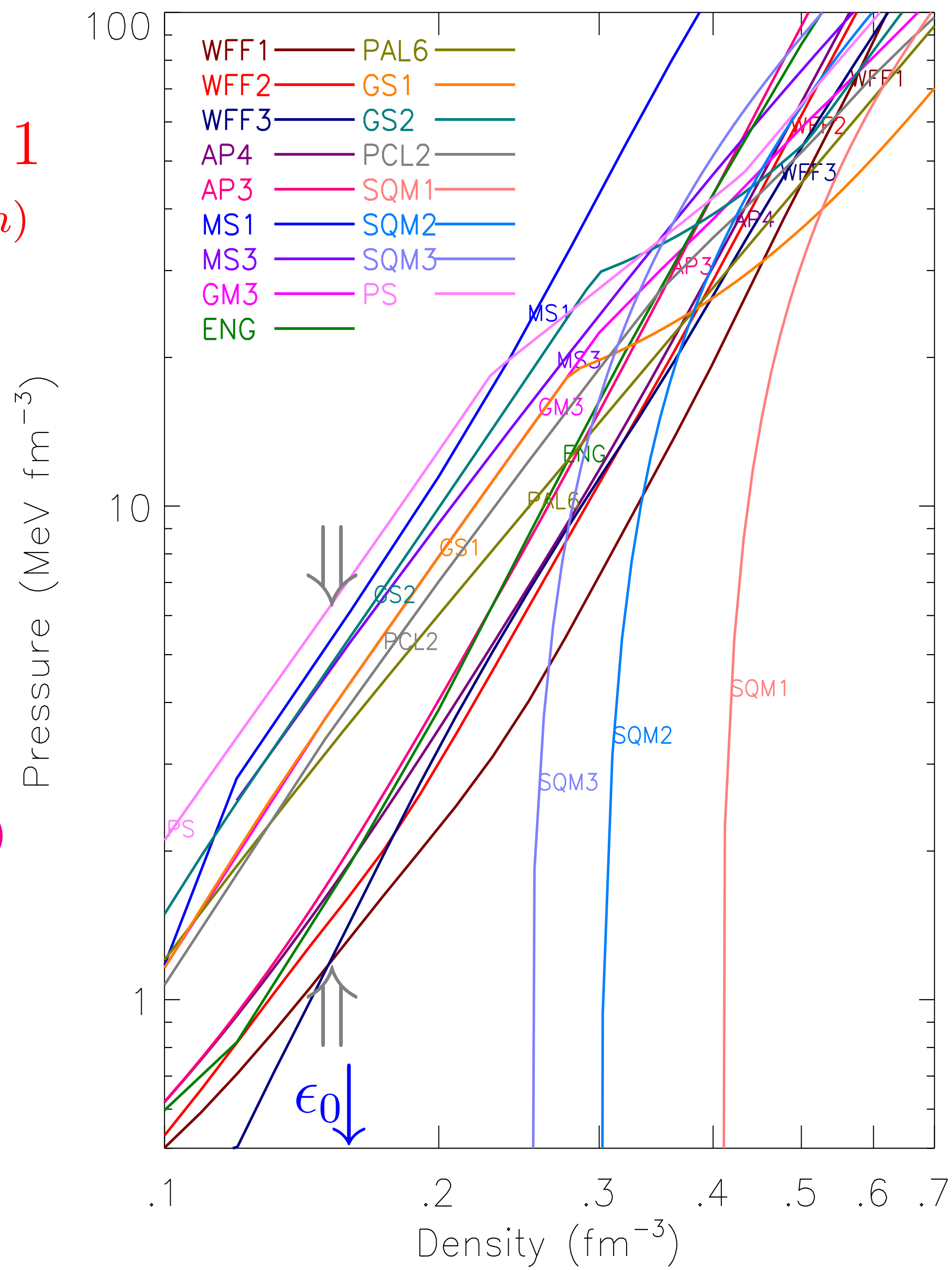
Wide variation:

$$1.2 < \frac{p(\epsilon_0)}{\text{MeV fm}^{-3}} < 7$$

GR phenomenological result (Lattimer & Prakash 2001)

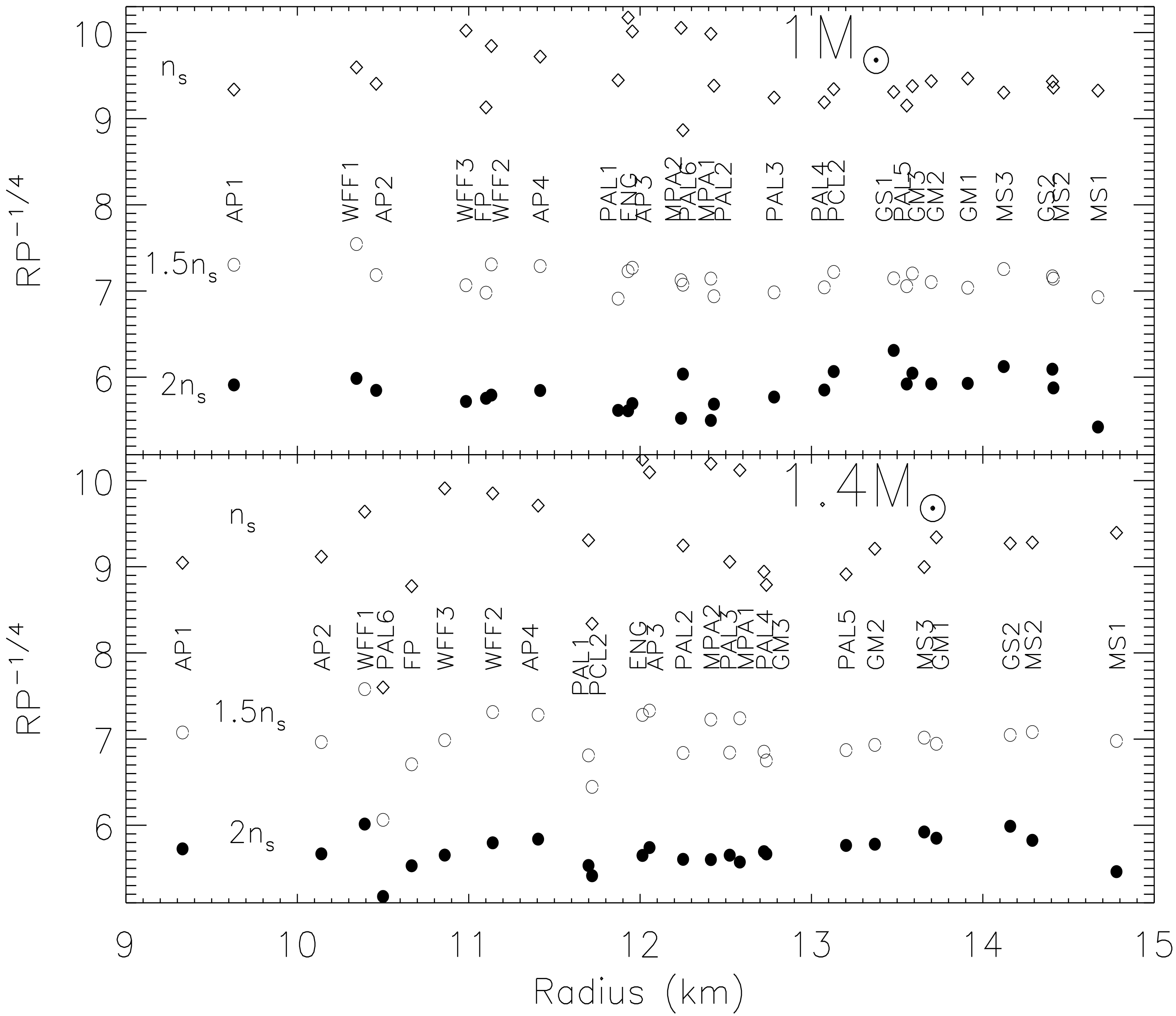
$$R \propto p_*^{1/4}\epsilon_*^{-1/2}$$

$$p_* = n^2\frac{dE_{sym}}{dn} = \frac{n^2 L}{3n_s}$$



The Radius – Pressure Correlation

$R \propto p^{1/4}$



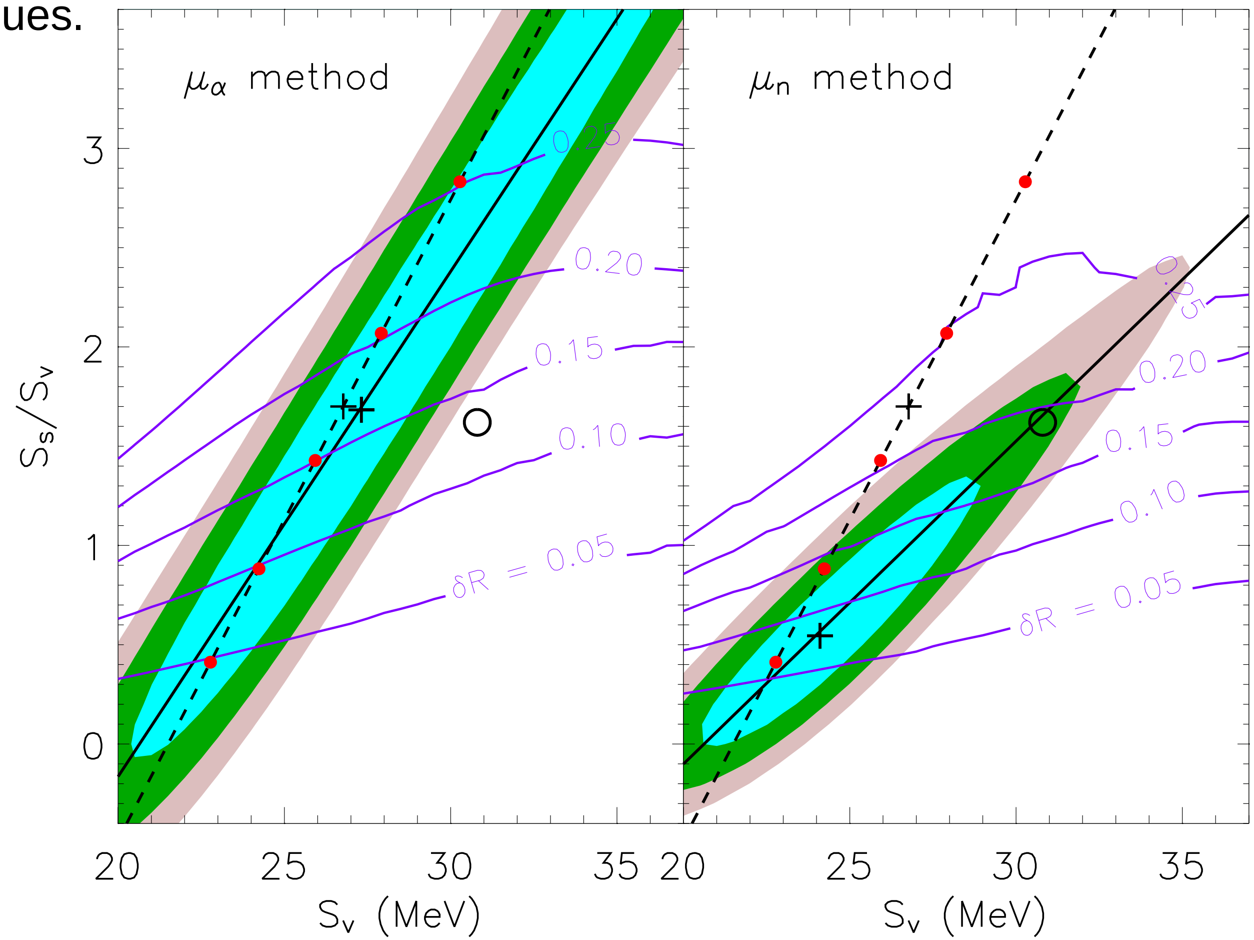
Lattimer & Prakash (2001)

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_C Z^2 A^{-1/3}$$

Optimizing to energies of nuclei yields a strong correlation between S_v and S_s, but not highly significant individual values.



Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)

Crosses: Best fits

Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)

δR is the predicted neutron skin thickness of Pb^{208} (fm)

Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Redshifts from Pulse Shape Modulation*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements
  $\beta \equiv GM/Rc^2$

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_\infty^2 T_\infty^2}$$

- Redshift

$$z = (1-2\beta)^{-1/2} - 1$$

- Eddington flux

$$F_{EDD} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$

- Crust thickness

$$\frac{m_b c^2}{2}\ln\mathcal{H} \equiv h_t = \int_0^{p_t}\frac{dp}{n} = \mu_{n,t} - \mu_{n,t}(p=0)$$

$$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} = \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$

- Moment of Inertia

$$I \simeq (0.237 \pm 0.008)MR^2(1+2.84\beta+18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$

- Crustal Moment of Inertia

$$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$

- Binding Energy

$$\mathrm{B.E.} \simeq (0.60 \pm 0.05)\frac{\beta}{1-\beta/2}$$

Radiation Radius

- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

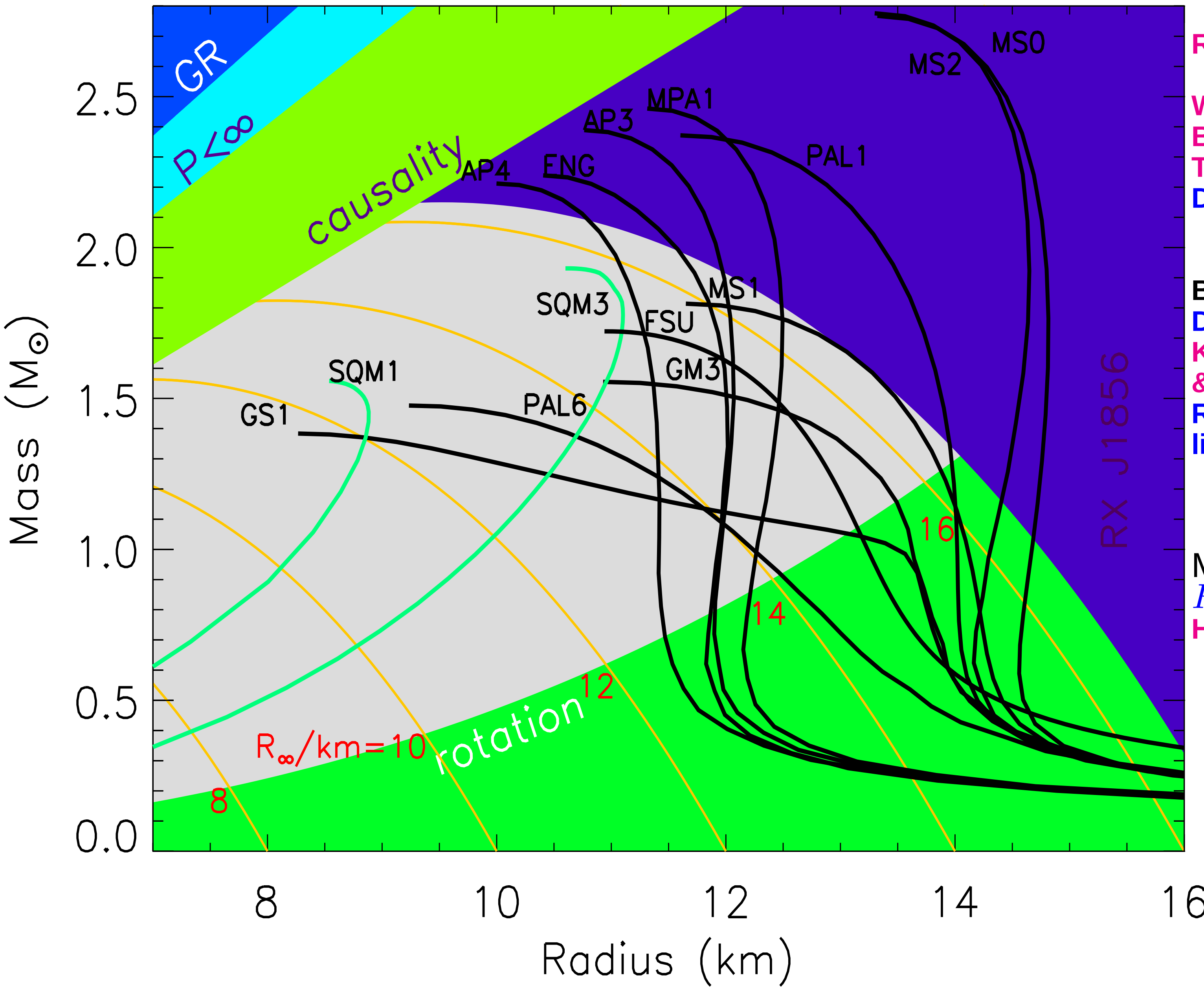
- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)

    However, large implied $R_\infty > 17$ km for RX J1856-3754

  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)
  - X-ray pulsars in systems of known distance

CXOU J010043.1-721134 in SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

Radiation Radius: Nearby Neutron Star



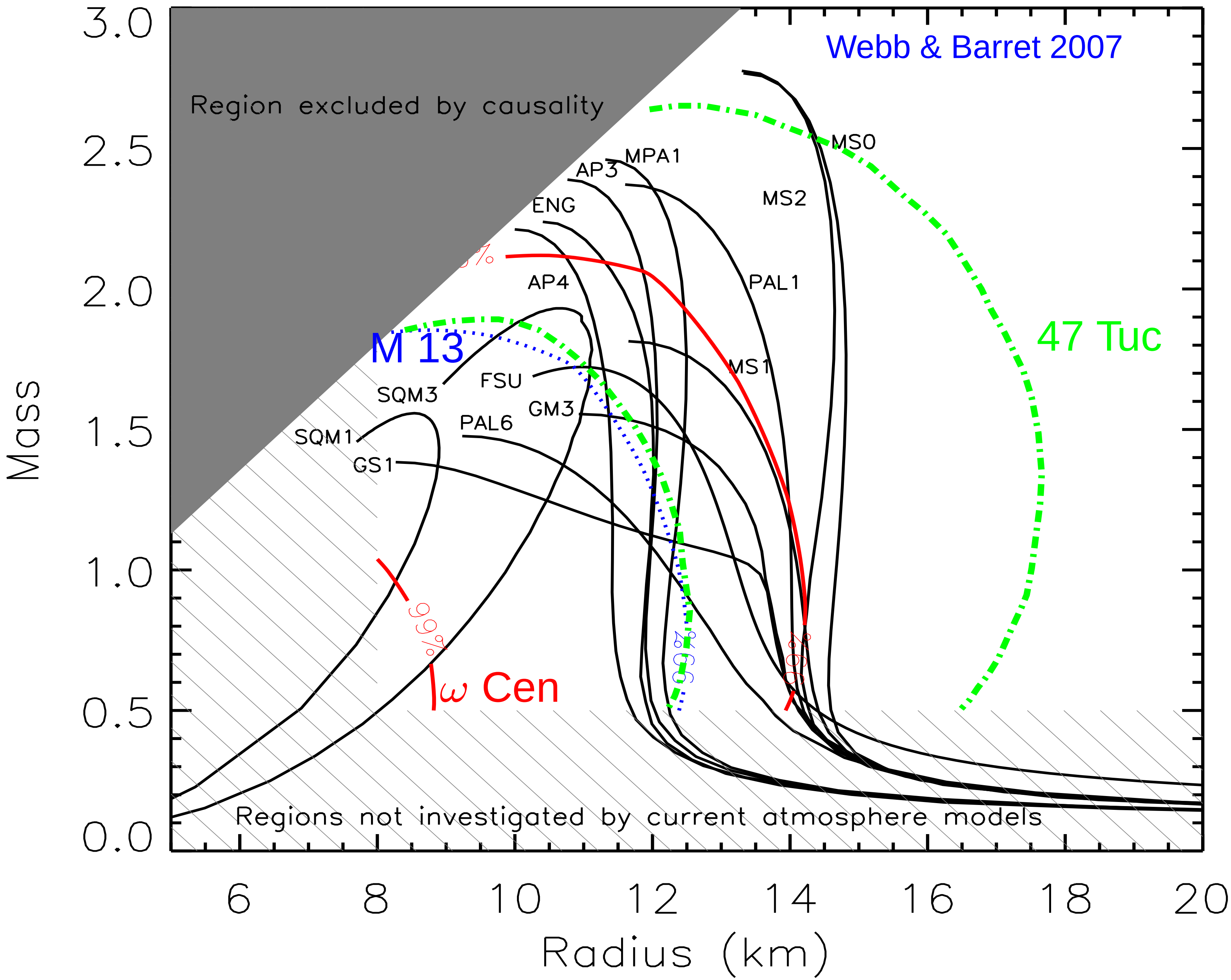
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk
& Anderson 2002
Raises R_∞
limit to 19.5 km

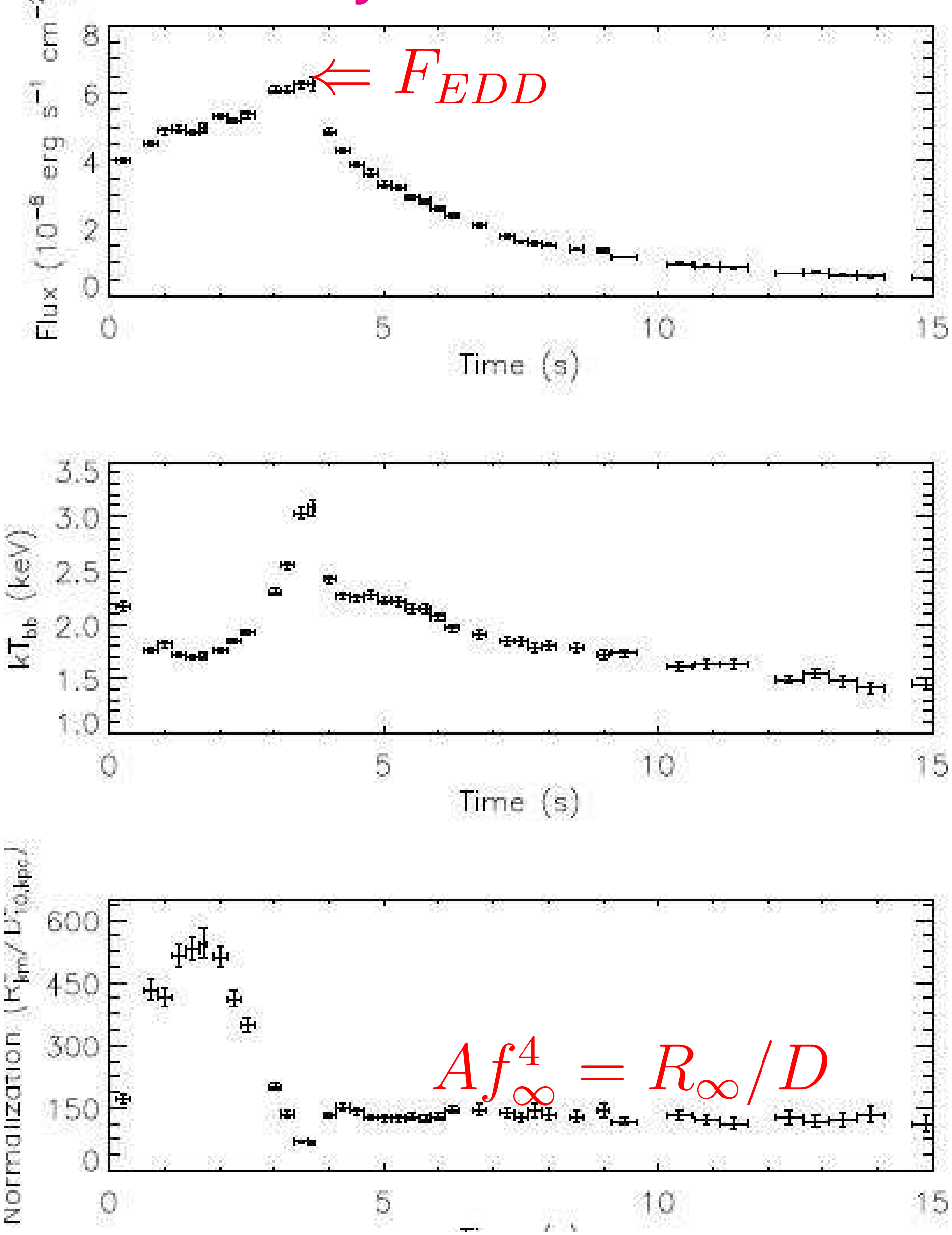
Magnetic H atmospher
$R_\infty \approx 17$ **km**
Ho et al. 2007

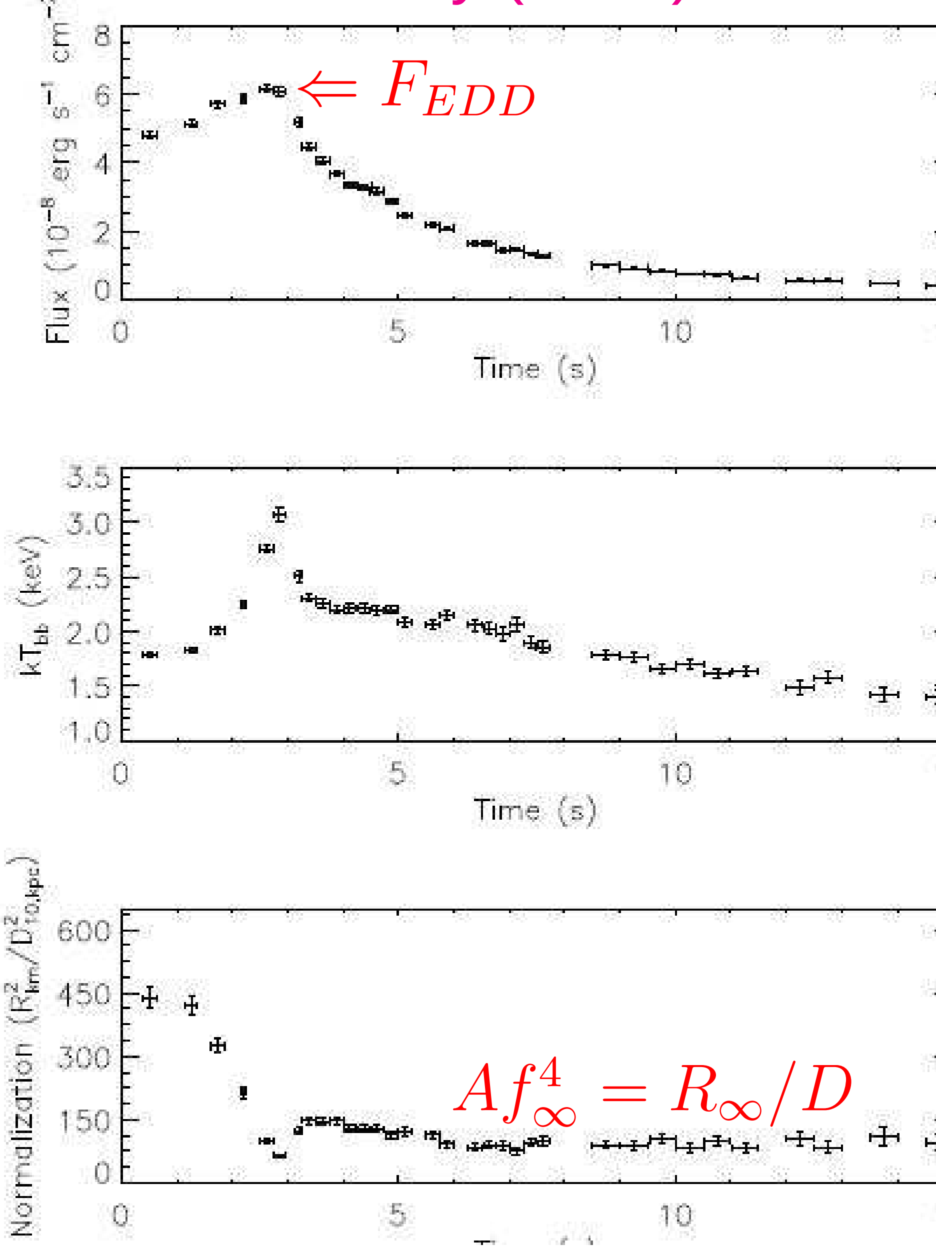
Radiation Radius: Globular Cluster Sources



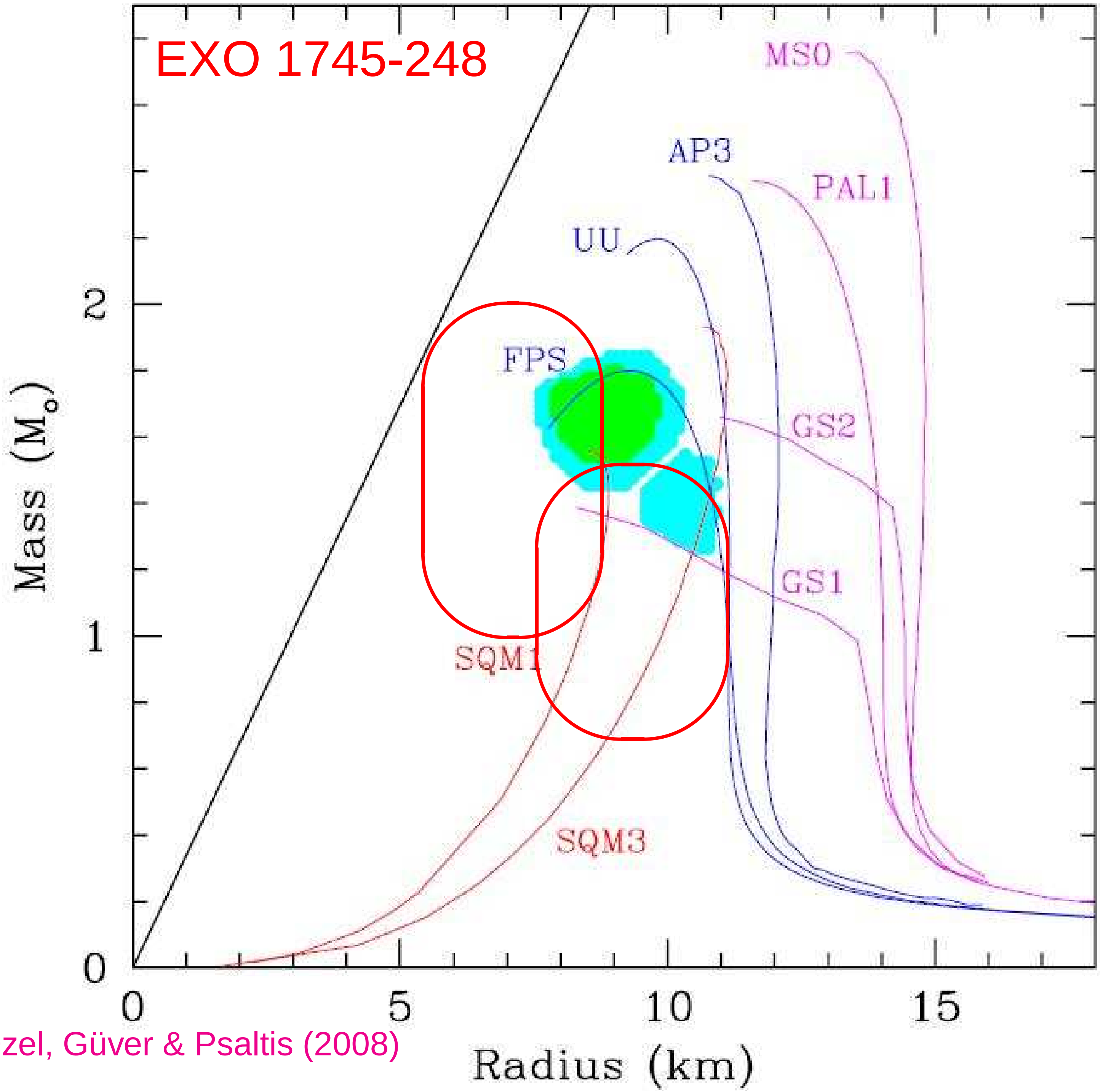
Cooling Following An X-Ray Burst

Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)





EXO 1745-248

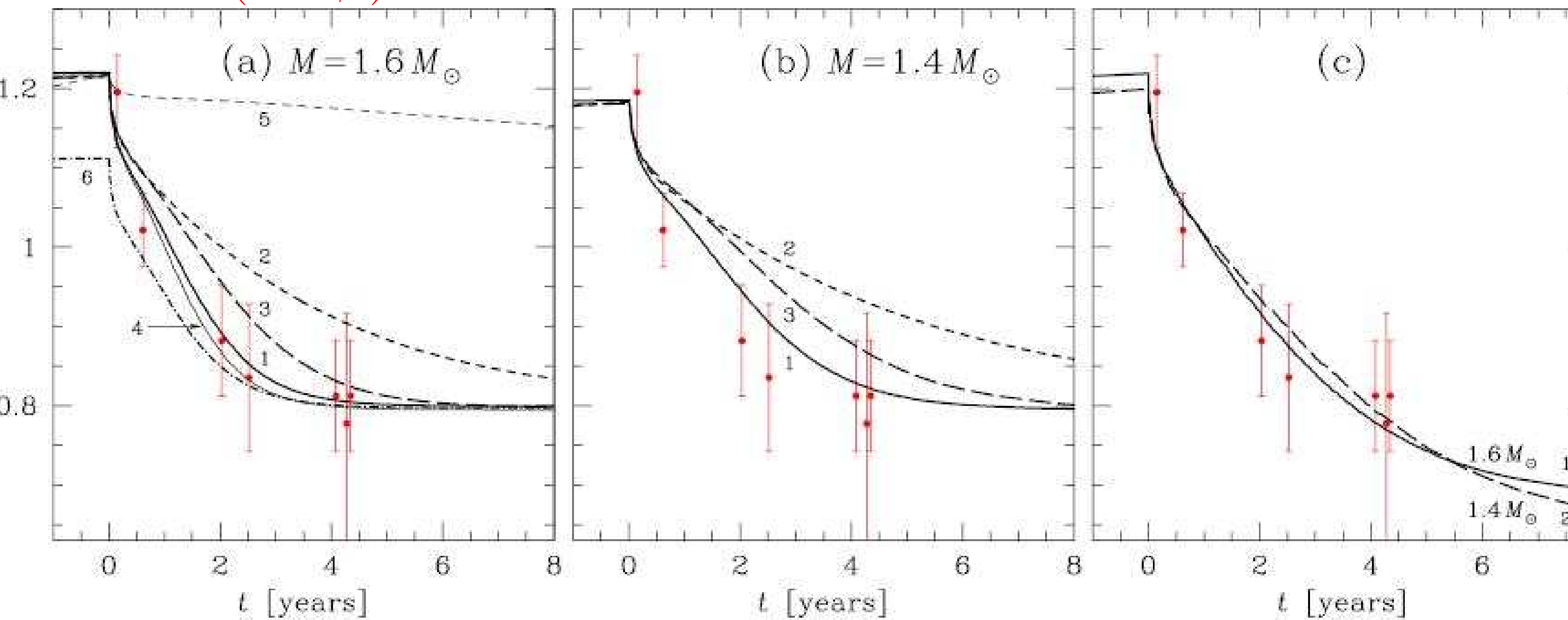


Özel, Güver & Psaltis (2008)

Crustal Heating in X-Ray Transients

Observations:
Cackett, Wijnands, Linares, Miller, Homan & Lewin (2006)

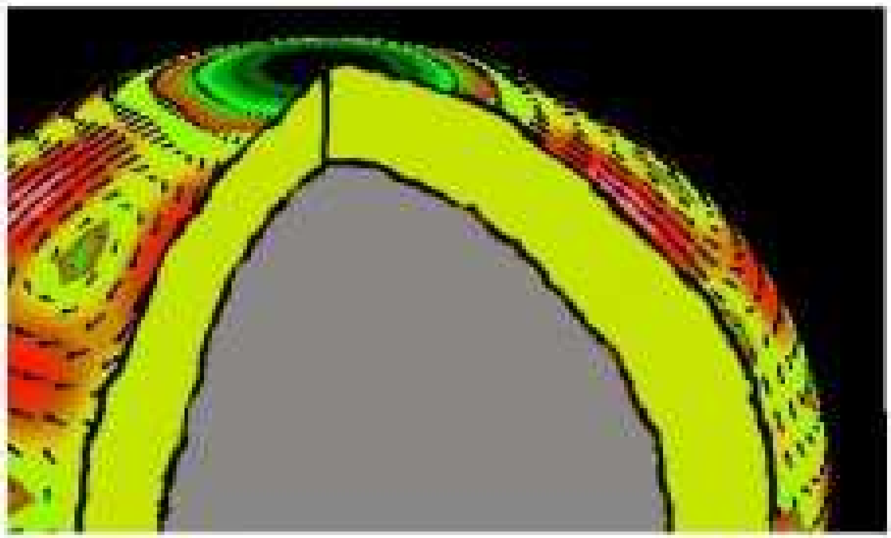
$$\tau \propto \frac{C_V \Delta^2}{\kappa(1-2\beta)^{3/2}} \propto \frac{C_V (1-2\beta)^{1/2}(\mathcal{H}-1)^2 R^4}{\kappa M^2}$$

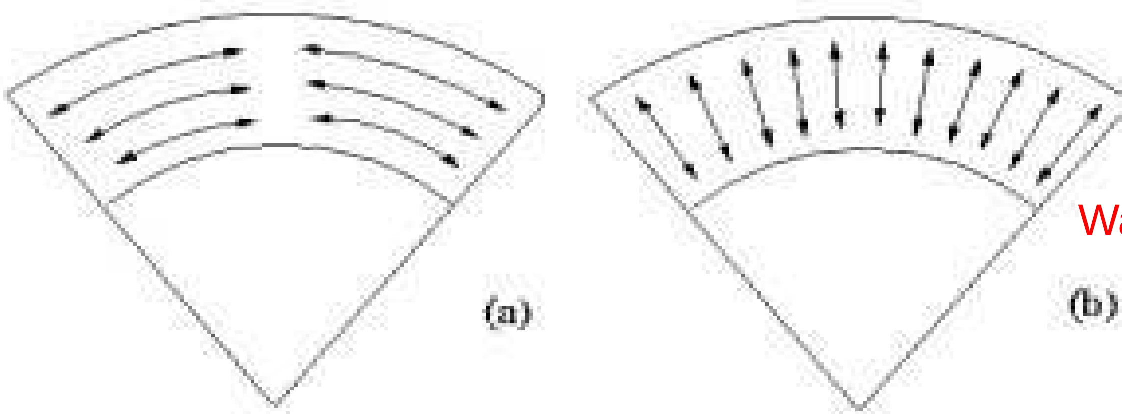


Shertnin, Yakovlev, Haensel & Potekhin (2007)

Giant Flares in Soft Gamma-Ray Repeaters (SGRs)

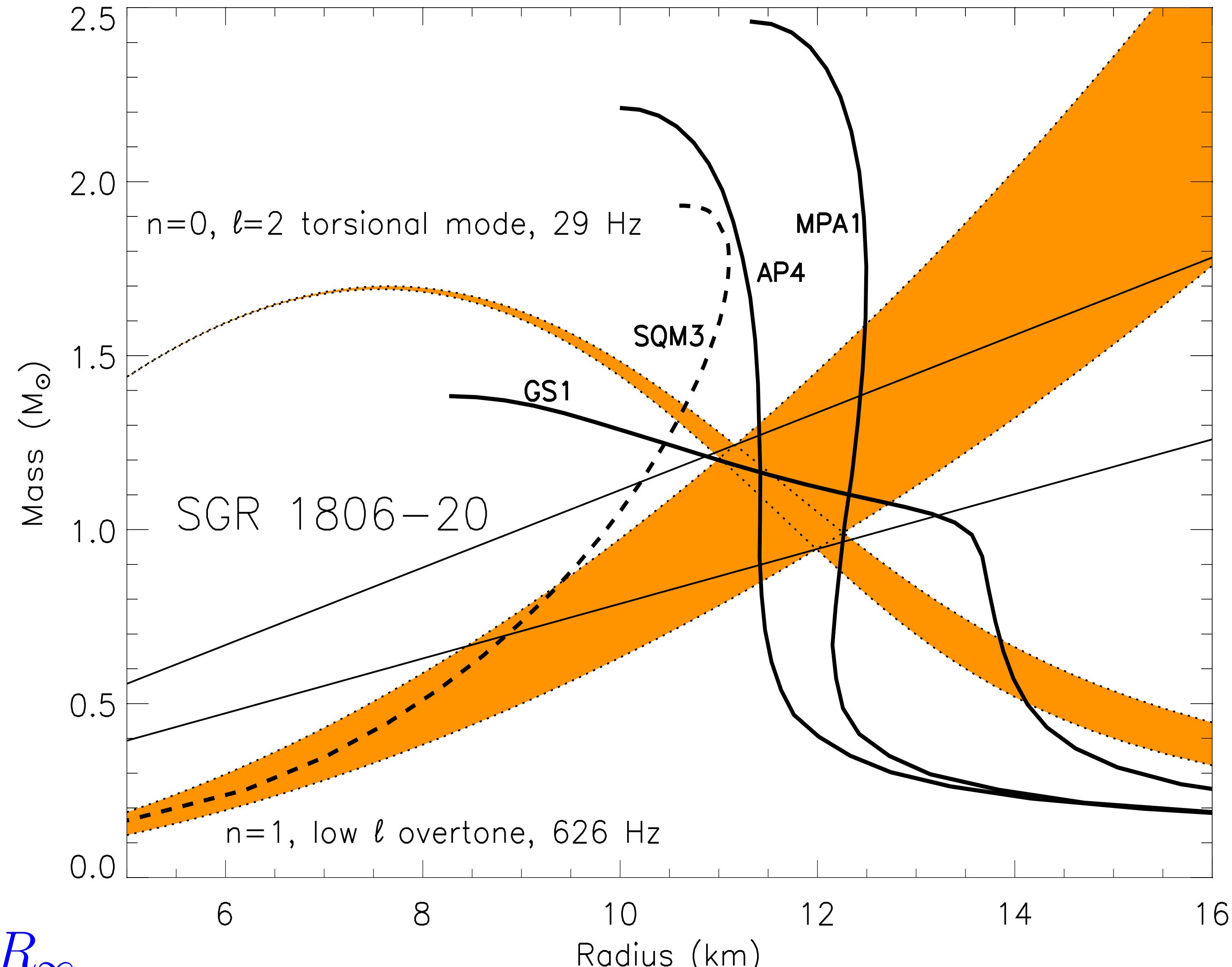
Quasi-periodic oscillations observed following giant flares in three soft gamma-ray repeaters (Israel et al. 2005; Strohmayer & Watts 2005, 6; Watts & Strohmayer 2006) which are believed to be highly magnetized neutron stars (magnetars).
Fields decay and twist, becoming periodically unstable. Eventually, the field lines snap and shift, launching starquakes and bursts of gamma-rays. Torsional shear modes are much easier to excite than radial modes.





Watts & Reddy (2006)

Neutron Star Seismology



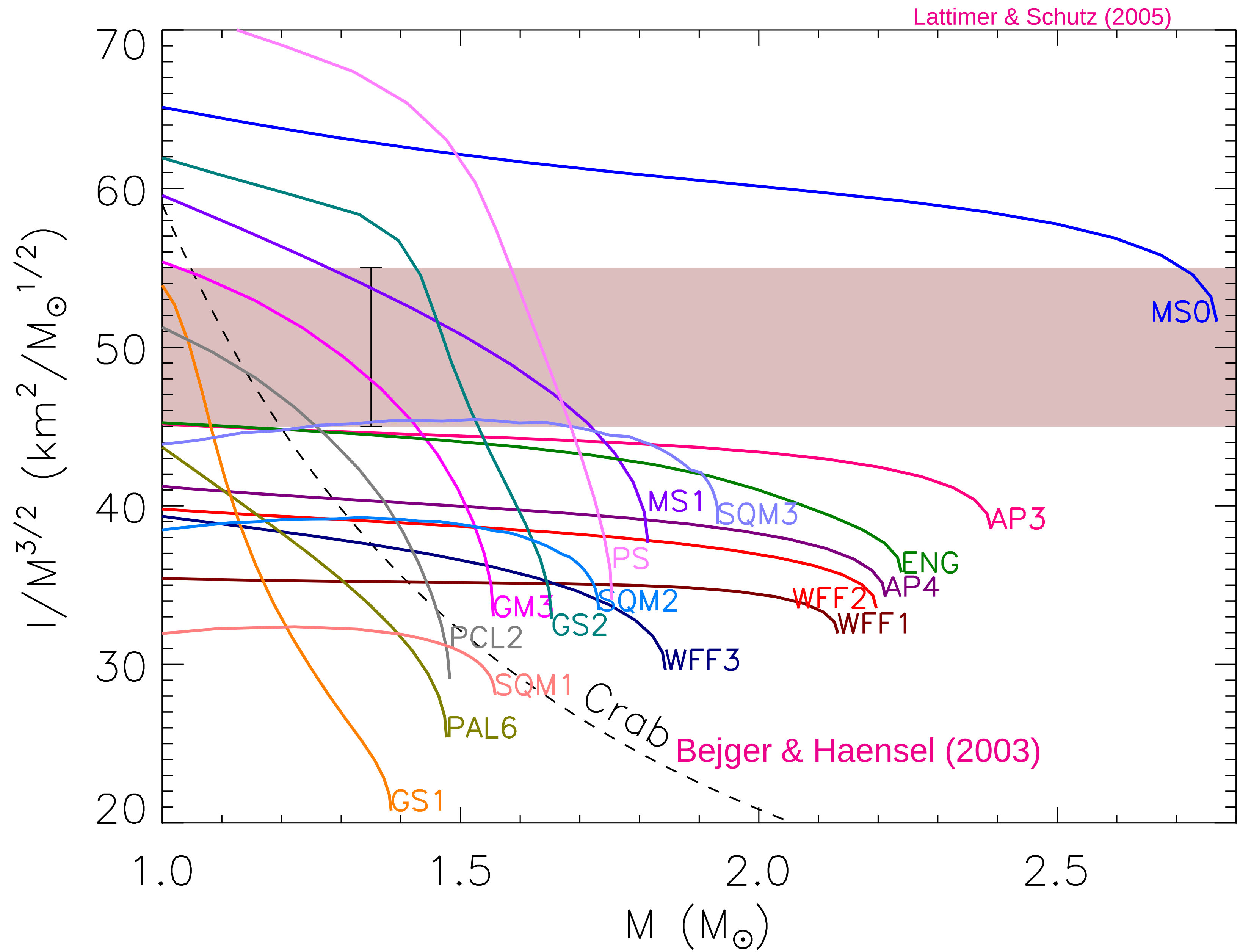
$$f_{n=0} \sim v_t / R_\infty$$
$$f_{n>0} \sim v_r \frac{1-2\beta}{\Delta} \sim v_r \frac{M}{R^2(\mathcal{H}-1)}$$

Strohmayer & Watts (2005)
Samuelsson & Andersson (2006)
Lattimer & Prakash (2006)

Moment of Inertia

- Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- Precession alters inclination angle and periastron advance
- More EOS sensitive than R: $I \propto MR^2$
- Requires extremely relativistic system to extract
- Double pulsar PSR J0737-3037 is a marginal candidate
- Even more relativistic systems should be found, based on dimness and nearness of PSR J0737-3037

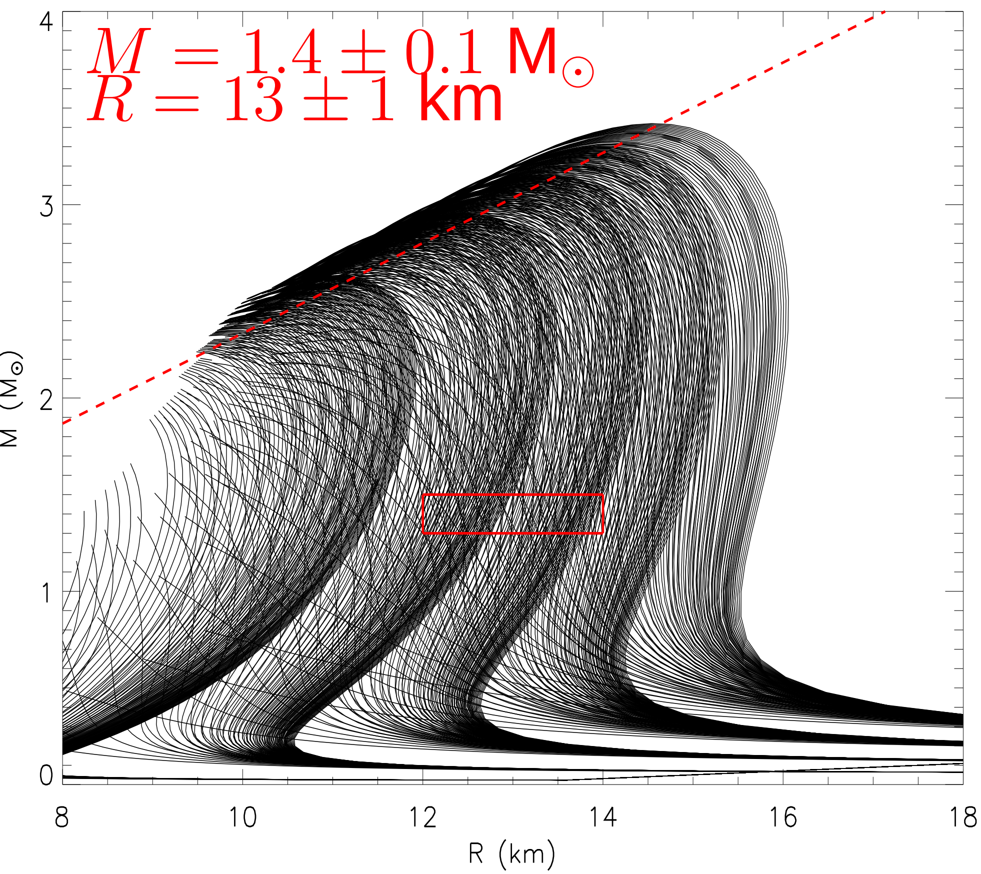
EOS Constraint

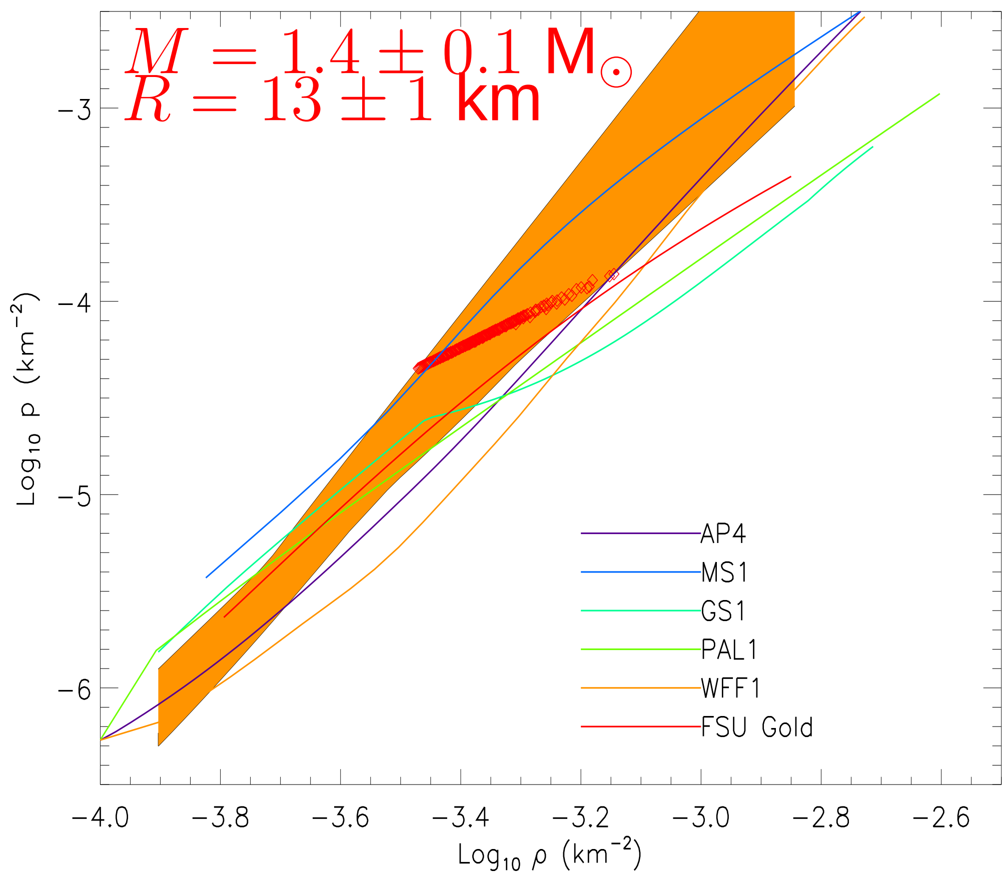


TOV Inversion

How would a simultaneous $M - R$ determination constrain the EOS? Each M-R curve specifies a unique $p - \rho$ relation.

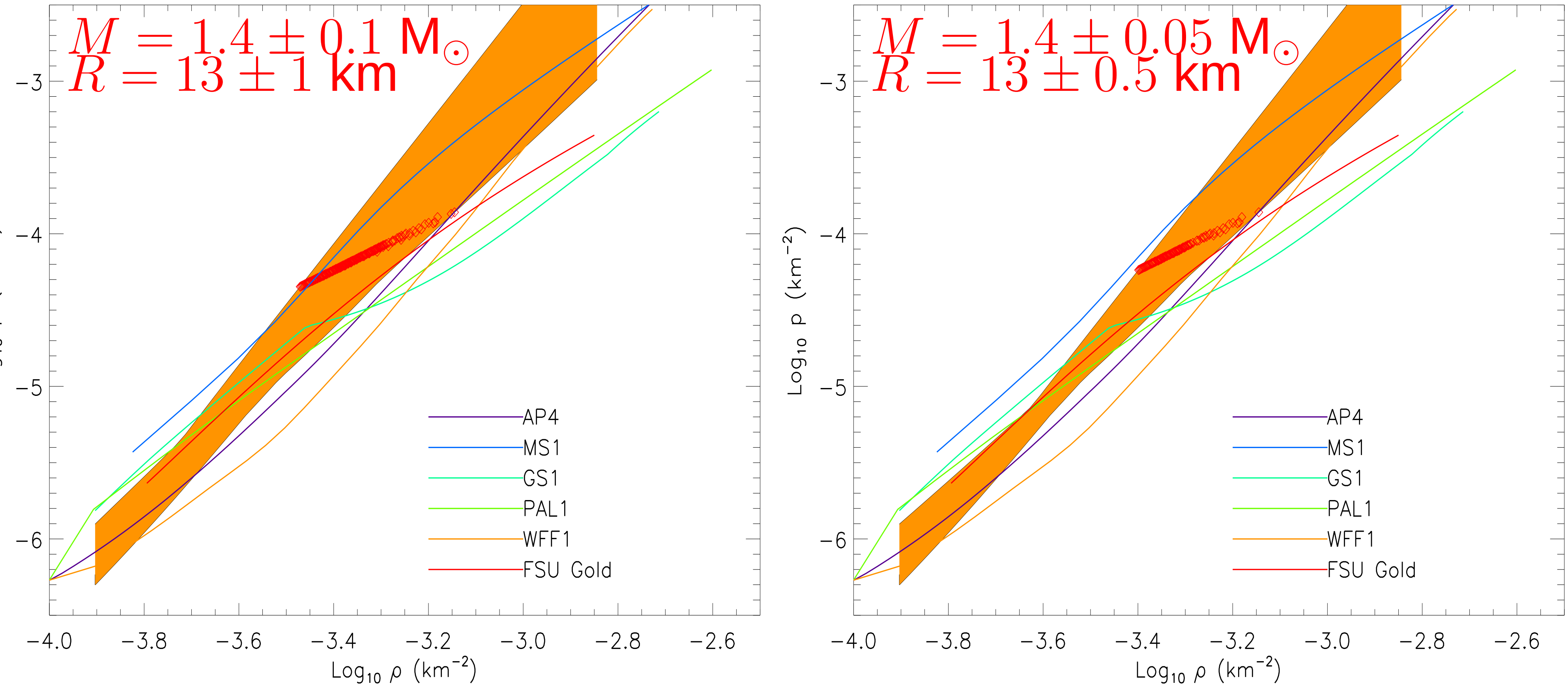
- Generate physically reasonable $M - R$ curves and the $p - \rho$ relations that they specify.
- Generate arbitrary $p - \rho$ relations and compute $M - R$ curves from them; select those $M - R$ curves passing within the error box.





TOV Inversion (cont.)

Dependence on measurement errors



The current uncertainty in the subnuclear EOS introduces significant width to the inferred high-density pressure-density relation.

Conclusions

- **Neutron stars are a powerful laboratory to constrain dense matter physics, especially the symmetry energy and composition at supranuclear densities.**
- **Many aspects of neutron star structure depend on specific equation of state parameters or their density dependence in a model-independent fashion.**
- **Increasing evidence supports the existence of massive neutron stars ($M \gtrsim 1.7$ $M_\odot$), constraining exotic matter.**
- **Many kinds of observations are now available to constrain neutron star radii, although no reliable measures yet exist.**
- **An accurate, simultaneous mass and radius measurement from even one neutron star would provide a significant constraint.**